Optimal Screening with Securities

Nicolas Figueroa
Pontificia Universidad Catolica de Chile

Nicolas Inostroza
University of Toronto*

August 1, 2023

Abstract

A liquidity-constrained asset owner designs an asset-backed security to raise funds from an informed liquidity supplier. Information-insensitive securities reduce the liquidity supplier's information rents. The optimal screening mechanism with financial securities consists of a *debt menu* with face values monotonically ordered in the liquidity supplier's valuation. We leverage this characterization to show that when the liquidity supplier's private information becomes more accurate (Lehmann [1988]), the issuer optimally offers debt securities with smaller face values. Surprisingly, the concavity of debt on the asset's future cashflows implies that the issuer may benefit from trading with a more informed liquidity supplier. Our results challenge the conventional notion that, when trading securities, the informed party should obtain an information-sensitive security and suggest a novel rationale for the emergence of venture debt and the prevalence of collateralized lending.

JEL classification: D82, D83, G39.
Keywords: Security Design, Informed Investors, Information Rents, Information Sensitivity, Lehmann order.

*Emails: nicolas.inostroza@rotman.utoronto.ca, nicolasf@uc.cl. For comments and useful suggestions, we thank Heski Bar-Isaac, Ing-Haw Cheng, Rahul Deb, Felipe Iachan, Camille Hébert, Hao Li, Lucas Maestri, Humberto Moreira, Nicola Persico, Luciano Pomatto, John Quah, Uday Rajan, Doron Ravid, Xianwen Shi, Anton Tsoy, Paul Voss, Asher Wolinsky and seminar participants at various conferences and institutions where the paper was presented. The usual disclaimer applies. Nicolás Figueroa gratefully acknowledges financial support from ANID PIA/APOYO AFB220003

1 Introduction

Financial institutions often sell securities to raise funds and fulfill their short-term obligations. In such transactions, liquidity providers typically possess valuable private information regarding the securities' underlying assets. However, which type of securities should a liquidity-constrained institution sell when facing informed investors? Despite the prevalence of economic environments where the buy-side has superior private information (e.g., venture capital funds, management buyout groups, restructuring teams, publishers, etc), the primitive question of how to effectively screen investors endowed with superior private information using financial securities remains unresolved. This paper tries to close this gap and provides some novel insights.

The heterogeneity in investors' valuations for financial assets is prevalent in financial markets.[1] This heterogeneity can originate from multiple sources. It may emerge, e.g., as a response to discriminating tax rules, as the result of asymmetric information among different market participants, as the output of heterogeneous technologies to process private and public information, or as the result of exposures to idiosyncratic, nontradeable risks. In this paper, we study how an asset owner can screen an investor's private information by optimally choosing the security design.

To illustrate, consider a startup trying to raise funds from a venture capital fund (VC). VCs specialize in funding and coaching similar projects, and they usually have superior information about the potential for growth and the future cashflows that could be generated.[2] Startups, on the other hand, struggle to raise cash to fund their initial operations and are therefore strongly liquidity-constrained. To raise liquid funds, entrepreneurs usually sell claims on the startup's future cashflows (i.e., securities) in exchange for cash. If the entrepreneur could design the securities to be sold to the VC to maximize the amount of funds raised, which security would she choose?

This paper addresses a key theoretical question in financial economics: how to optimally screen investors endowed with superior private information using flexible financial securities. The main insights from the existing theoretical literature focus largely on the case where the asset owner is the one endowed with private information. The standard intuition there suggests that the informed party should keep an information-sensitive security, e.g., an equity

[1]Bagwell [1991] and Bagwell [1992] document investor heterogeneity in the context of stock repurchases; Bradley et al. [1988] find evidence of heterogeneity in the context of corporate acquisitions. Bernardo and Cornell [1997], in turn, extend the analysis to the case of complex derivatives.

[2]Several papers find evidence of VC firms repeatable skills. See, e.g., Kaplan and Schoar [2005] and Ewens and Rhodes-Kropf [2015]. The performance persistence might originate from access to networks (Hochberg et al. [2007]), high levels of industry experience (Hellmann and Puri [2002]), or screening skills (Sorensen [2007]).

stake or a call option.[3] We demonstrate, however, the existence of a fundamental rationale for the issuer to offer information-insensitive securities to informed investors. By doing so, the issuer can screen the investor's private information more effectively and reduce the latter's information rents.

To gain some intuition, consider the following example. There is an issuer (she) and a liquidity supplier (he) who can be of two types, namely, either H or L. Assume that H's beliefs about the asset's future cashflows allocate more probability mass to higher realizations relative to the issuer, whereas L's beliefs allocate less probability mass to higher realizations than the issuer (formally, H's beliefs dominate the issuer's beliefs, which in turn dominate L's beliefs according to the monotone likelihood ratio order (MLR)). This could be either because the asset is more (less) productive when managed by type H (type L), or simply because H (L) is more (less) optimistic about the asset's prospects. We show that when the issuer is subject to liquidity constrains, she is better off by selling debt instruments.

To see the intuition, suppose that the issuer designs a menu of securities and respective payments, $\left\{\left(s^L, p^L\right), \left(s^H, p^H\right)\right\}$, where s^i represents the security and p^i represents the payment designed for type $i \in \{H, L\}$. The issuer can replicate the expected cashflows of s^L with a debt contract, s^L_d, which promises fixed payout $d > 0$ and seniority if this amount is not met. If the face value d is chosen so that L is indifferent between s^L and s^L_d, then type H and the issuer value s^L_d strictly less than s^L. Indeed, the latter assign more probability mass (relative to L), to high cashflow realizations. For these realizations, however, s^L_d always offers the same flat payout, d. In other words, s^L_d minimizes the upside, which are also the states that type H and the issuer deem more likely relative to L. Thus, by changing the original security s^L for the debt contract s^L_d, the issuer reduces the *information rents* that she needs to leave to type H to prevent him from mimicking type L. The issuer can therefore increase the price charged to H for security s^H, without spoiling his incentives. Furthermore, the issuer values the security she sells to type L, s^L_d, strictly less than s_L, and therefore she keeps a larger share of the future cashflows. Thus, by designing a debt security for the low type, the issuer both raises more funds and at the same time sells smaller securities.

The heuristic described above is general in that it can be extended to the case with an *arbitrary* number of liquidity supplier's types. Using a replication argument, we show that any incentive compatible and individually rational mechanism can be dominated by another

[3] There are different rationales for this. In Nachman and Noe [1994], keeping information-sensitive security increase the issuer's skin in the game and are therefore a form to costly signal her private information. In DeMarzo and Duffie [1999], Biais and Mariotti [2005], the optimal strategy involves selling debt as a commitment device for the issuer not to exploit her future private information when trading with the liquidity supplier. Debt offers a fixed payout unaffected by cashflow outcomes during favorable times, while providing maximum downside protection.

mechanism where all types purchase debt securities. The optimal mechanism consists of a *menu of debt contracts*, with face values monotonically ordered in the liquidity supplier's type. That is, the more optimistic liquidity supplier types purchase larger amounts of debt. Our main technical insight is that any menu of securities (with their associated prices) can be dominated by modifying the original securities for debt securities while guaranteeing that the incentive constraints are not spoiled either locally or globally. Indeed, with more than two types, the intuition provided above may fail as the incentives of low types to mimic high types are exacerbated when the latter are offered debt securities. We show that the securities in any incentive compatible mechanism must satisfy a property analogous to the monotonicity condition in the one-dimensional case. We leverage this property to show that by correctly permuting the securities for debt contracts, all incentive constraints are in fact relaxed. This last observation allows the issuer to increase the price of the securities being sold and hence maximizes her revenue.

We note that the heuristic described above does not contradict the main insights in the security design literature that postulate that more informed agents should expose themselves relatively more to the asset's cashflows to signal their private information (Leland and Pyle [1977], Ross [1977], Myers and Majluf [1984], etc.). In fact, under the optimal mechanism, more optimistic liquidity supplier types purchase larger fractions of the underlying asset (i.e., debt with higher face values), which obviously expose them to right-tail risk. Perhaps surprisingly, however, we find that exploiting the information sensitivity of securities to screen the liquidity suppliers private information is completely ineffective and is strictly dominated by information-insensitive instruments.

Following a direct mechanism design approach, we extend the classical results in the optimal screening literature to environments with a rich allocation space in which the issuer has the flexibility design financial securities, infinite-dimensional objects.[4] One of the technical challenges we face in extending the classical results to our environment is that we lose the structure usually assumed in those earlier models.[5] We provide a full characterization of the mechanism design problem with securities. The advantage of this approach vis-à-vis the former results in the literature is that it allows us to find analytical expressions for the issuer's expected revenue and the liquidity supplier's information rents. This characterization

[4]The classical literature focuses on the case where the issuer either decides whether to sell the whole asset to the liquidity supplier, potentially in a stochastic manner. This is equivalent to asset selloffs or equity stakes. Instead, we propose enriching the allocation space to encompass all types of securities (e.g., debt, options, arbitrary tranches, etc.).

[5]It is standard to assume that the buyer's payoff has increasing differences in the allocation and the buyer's type. This assumption, together with some regularity conditions, jointly imply that local incentive constraints imply the global constraints. There is no obvious extension of this property to the infinite-dimensional space of all securities.

is instrumental to derive novel monotone comparative statics results relating the underlying information structure and the issuer's ability to raise external financing.

We show that when the liquidity supplier's private information about the asset's future cashflows becomes more *accurate* (Lehmann [1988]), the issuer optimally responds by selling smaller debt securities. Intuitively, when the liquidity supplier's information improves, the issuer is forced to give up more information rents to the liquidity supplier. To minimize the incentives of high types to mimic low types, the issuer truncates the securities designed to the low types at a lower level. Perhaps surprisingly, however, this does not mean that the issuer's ability to raise external funding deteriorates when facing investors endowed with more accurate signals. We document a novel countervailing effect associated with the *geometry* of the optimal securities. The concavity of debt on the asset future cashflows, implies that improving the accuracy of the liquidity supplier's information increases the liquidity suppliers valuation for these securities. This effect is similar to a reduction in uncertainty for a risk-averse agent, which increases the utility she derives from the security. We show by means of an example that under some conditions, the second effect prevails, leading to the striking conclusion that an issuer with the ability to flexibly design financial securities may benefit from facing more informed investors, whereas the same issuer constrained to sell linear instruments as in the tradition of the screening literature (e.g., the whole asset, equity stakes) suffers from trading with the more informed investors. Our results emphasize the stark differences in the economics of optimal screening between financial products and other types of assets, and call for prudence when extrapolating economic intuitions. The characterization of optimally-designed financial mechanisms appears to be critical in understanding the efficiency gains arising from enhanced transparency in financial markets.

We argue that accuracy (Lehmann [1988]) is an appealing notion of informativeness in our environment for three reasons. First, provided that signals have the MLR property, accuracy is less restrictive than the Blackwell ordering in that it compares more signal structures.[6] Furthermore, accuracy implies the standard notion of informativeness usually assumed in the information economics literature.[7] Second, the concept of accuracy is tightly related to the idea of interdependence. When an experiment is more accurate, the comovement between fundamentals and signals becomes stronger. Finally, the Lehmann ordering compares experiments (i.e., conditional distributions) as opposed to comparing joint distributions. Intuitively,

[6] Indeed, provided that signals satisfy the MLRP, any two signals ordered according to Blackwell are also ordered according to Lehmann [1988]. When the state space is binary, both notions of informativeness coincide (see, e.g., Jewitt [2007]). More recently, Kim [2022] showed that Lehmann domination is closely related to the concept of quasigarbling, a generalization of information garbling.

[7] Fixing the prior example, if two experiments are ordered according to Lehmann, then the distribution of posterior estimates induced by the experiments' signals are ranked in convex order. See Proposition 2 below.

we want to change the quality of the liquidity supplier's information without changing the distribution of the asset's future cashflows or the distribution of liquidity supplier types, as this would change the underlying economic environment. We show that starting from a fixed (marginal) distribution of the assets' cashflows, one can increase the accuracy of the liquidity supplier's private information while preserving the (marginal) distribution of the liquidity supplier's private signals. Combined with our first observation, this implies that increasing signals' accuracy increases the liquidity supplier's private information about the fundamentals without changing the quality of underlying asset or the ex-ante distribution of liquidity supplier types, thus making the comparative statics exercise coherent. To the best of our knowledge, this is the first paper to perform monotone comparative statics using information orders in the context of security design.

Interestingly, the prediction that liquidity-constrained asset owners use debt instruments to raise funds from informed investors is consistent with some empirical regularities. In the case of VC funding, our predictions are consistent with the emergence of venture debt. Indeed, according to Tykvová [2017], approximately one-third of the current venture-backed companies use debt instruments to raise funds. A plausible explanation for this new trend is that as VC funds become more competitive, they retain lower information rents. Concomitant with the emergence of venture debt is the fact that VCs with a founder-friendly reputation have gained prominence in the last decade (Ewens et al. [2018], Lerner and Nanda [2020]). Contrary to the former governance approach, which entailed the intensive monitoring of startups, VCs are adopting a hands-off approach, leaving much discretion to the entrepreneurs. These regularities seem consistent with the idea that startups have greater bargaining power vis-à-vis VC funds. Our theory shows that such startups can limit the VCs' information rents by choosing venture debt financing.

In the context of structured financial products, such as the market for mortgage-backed securities (MBS), where the buyers of these products are generally large investment banks, brokerage firms, and institutional investors who typically have substantial expertise in valuing the securities through their knowledge of secondary market conditions and their access to proprietary valuation models.[8] Our predictions are broadly consistent with the prevalence of tranching in securitization, wherein investors are promised a fixed face value and payments contingent on the underlying assets' cashflows when the face value is not met. Finally, our predictions are also consistent with the experience of the Resolution Trust Corporation (RTC) and the FDIC, which are institutions in charge of disposing the assets of failed financial institutions. These institutions typically face investors with superior information about the

[8] Bernardo and Cornell [1997] analyze data from an auction of collateralized mortgage obligations (CMO) and find statistical evidence of a large dispersion in investors' valuation for the securities.

specific assets. Their objective is to maximize the proceeds of these sales to pay back the original stakeholders of the failing institutions. Using pooled assets auctions and securitized vehicles, they have dramatically increased the funds raised from these sales.

Our findings concerning the relationship between the informational environment and the issuer's capacity to raise external financing, contributes valuable insights to the ongoing public debate regarding the advantages and disadvantages of altering information dissemination environment in financial markets. In practice, the accuracy of liquidity suppliers' information is influenced by various factors, such as the introduction of new regulations, like Regulation AB, which imposes disclosure requirements for asset-backed securities offerings. This disclosure enables liquidity providers to assess the security's value based on their unique preferences and beliefs. Additionally, advancements in technological tools used by market participants to process information and the due diligence efforts of liquidity suppliers can also influence the accuracy of of the available information. While our paper remains agnostic on the determinants of the underlying information structure and prevailing information asymmetries, our analysis establishes sufficient primitive conditions to identify how changes in investors' information accuracy could impact the financial institutions' funding capacity.

The rest of the paper is organized as follows. Below, we wrap up the introduction by discussing how our paper connects with the rest of the literature. Section 2 describes the primitives of the model. Section 3 contains the derivation of our first result establishing the optimality of *menus of debt contracts* and the characterization of the optimal mechanism. We further discuss the relation between securities' information sensitivity and the liquidity supplier's information rents. In section 4, we explore how changes in the accuracy of the liquidity supplier's private signal affects the optimal contracts, the amount of funds raised, and the agents' payoffs. All omitted proofs are relegated to the Appendix.

Related Literature

This paper relates to several strands of the literature. First, it contributes to the broad literature on security design under asymmetric information (see, e.g., Nachman and Noe [1994], DeMarzo and Duffie [1999], Biais and Mariotti [2005], DeMarzo and Fishman [2007], etc.).[9] We depart from those earlier models by assuming that when trading securities, it is the liquidity supplier who is endowed with private information. In the context of informed liquidity suppliers, DeMarzo et al. [2005] study general securities auctions with multiple ($N \geq 2$)

[9]Some recent work within this areas of study include Malenko and Tsoy [2020] and Lee and Rajan [2018]. They explore optimal security design under robust requirements.

bidders.[10] Following an indirect mechanism approach, they show that among all general symmetric mechanisms,[11] the first-price auction where buyers are restricted to bid call options (i.e., the buyer purchases a debt contract) maximizes the issuer's revenue. Burkart and Lee [2016] study a similar problem to ours but restrict the set of securities available to the issuer. Our result that the optimal mechanism consists of a debt menu is consistent with the intuition behind these papers. We provide a full characterization of the *unrestricted* solution to the liquidity-constrained issuer's problem with a single bidder. Following a direct mechanism approach, we show that the optimality of the menus of debt contracts originates on the fact that debt minimizes the liquidity suppliers' information rents and therefore allow the issuer to raise more funds. Our characterization is instrumental in performing novel comparative statics to shed light how changes to the primitives of the informational environment affect the optimal mechanism and the issuer's ability to raise external funding.

Axelson [2007] studies security auctions with multiple bidders when the issuer is liquidity-constrained. He finds that when the issuer restricts attention to a sealed-bid, uniform-price, K-units auction, debt is optimal. Liu [2016] follows a mechanism design approach similar to the one proposed in this paper and study optimal auctions when the issuer is constrained to sell equity securities. Liu and Bernhardt [2021] provide sufficient conditions under which equity plus cash auctions achieve optimality in the context of target-initiated takeovers. More recently, Yuan [2020] tackles a similar problem wherein multiple issuers compete by selling securities to informed liquidity suppliers. Building on the fact that competition among sellers leads to the winner's curse, she finds that there exists an equilibrium where all issuers sell debt securities. We show that the optimality of debt occurs even in the absence of competition. In our environment, the issuer has all the bargaining power and optimally designs a menu of debt contracts. We show that the optimality of debt originates from the fact that it allows the issuer to minimize information rents.

Our paper is also related to the emerging literature on information and security design. Yang [2020] studies a security design problem wherein the liquidity supplier can acquire costly information. He shows that a debt contract is uniquely optimal and minimizes the incentives to produce private information. In an informed issuer model, Daley et al. [2022] show that reducing the degree of asymmetric information between the issuer and the liquidity supplier leads the former to issue information-sensitive securities. Vanasco [2017], Szydlowski [2021] and Inostroza and Tsoy [2022] study the case where the issuer can design both the security

[10] They refer to auctions designed by the issuer as formal auctions. They also study games where the bidders have the freedom to bid with arbitrary securities, which they dub informal auctions.

[11] A general symmetric mechanism (GSM) is a symmetric incentive-compatible mechanism in which the highest type wins and pays a security chosen at random from a given set, S. The randomization can depend on the realization of types but not on the identity of the bidders.

and the information structure. Vanasco [2017] studies the case where issuer chooses both the security and the effort to improve the quality of the asset. She demonstrates that the adverse selection induced by the issuer's superior information mitigates the issuer's moral hazard problem when monitoring the quality of the pool of assets. Szydlowski [2021] shows that if the issuer's objective consists of raising a prespecified amount of funds, she is indifferent between all the securities yielding the same payoff. Inostroza and Tsoy [2022] show that when the issuer designs both the security and information structure, information-sensitive securities dominate debt instruments and pure equity maximizes the issuer's payoff.

The paper is also related to the literature on information orders and monotone comparative statics under uncertainty. Quah and Strulovici [2009] show that the Lehmann [1988] order is closely related to a natural order on utility functions, namely, the so-called interval dominance order. We show that the geometry of the optimal securities implies that information rents are ranked according to the interval dominance order and leverage this to show that the optimal mechanism is monotone in the Lehmann [1988] order. Persico [2000] and Ganuza and Penalva [2010] study the effect of increasing the accuracy of the agents' signals in auctions. Kim [1995] and Jewitt [2007] study the effect of increasing the informativeness of agents' signals in moral hazard problems, whereas Dewatripont et al. [1999] do so using career concerns model. More recently, Mekonnen and Vizcaíno [2022] have studied comparative statics of agents' optimal distributions of actions in Bayesian games when the informativeness of their signals increases. To the best of our knowledge, ours is the first paper to perform comparative statics in the context of optimal security design.

2 The Model

2.1 Security Design

The economy consists of an issuer (she) and a liquidity supplier (he). The issuer owns a risky asset that delivers a stochastic cashflow $\boldsymbol{y} \in \mathbb{R}_+$. There are two periods, $t \in \{1, 2\}$. In period 1, the issuer can sell a claim $s(\cdot)$ on the asset's period 2-cashflows to the liquidity supplier at a price p. In period 2, the asset delivers the stochastic cashflow $\boldsymbol{y} \in \mathbb{R}_+$, and the liquidity supplier obtains $s(\boldsymbol{y})$.

Securities. The issuer has the flexibility to design any arbitrary financial security satisfying limited liability and double-monotonicity in the asset's cashflows.[12] Therefore, the set

[12] These are standard assumptions in the security design literature (see, e.g., Nachman and Noe [1994]). When $s(y)$ is not monotone, the issuer can request (risk free) credit to a third party to boost cashflows and thus decrease the amount owed to liquidity supplier. In turn, when $y - s(y)$ is not monotone, the issuer can

of available securities is given by:

$$\mathcal{S} \equiv \{s : \mathbb{R}_+ \to \mathbb{R}_+ \quad \text{s.t:} \quad \begin{array}{l} (\text{LL}):\ 0 \leq s(y) \leq y,\ \forall y \geq 0 \\ (\text{M}) : s(y) \text{ and } y - s(y) \text{ are nondecreasing}\}. \end{array}$$

Information. In period 1, the liquidity supplier observes his type, $\boldsymbol{\omega} \in \{\omega_n\}_{n=1}^N$, which is private information. We denote by $F(\omega, y)$ the joint distribution of $\boldsymbol{\omega}$ and $\boldsymbol{y}$, and by Φ the marginal distribution of $\boldsymbol{\omega}$; for notational ease, we let $\Phi_n \equiv \mathbb{P}\{\boldsymbol{\omega} \leq \omega_n\}$ and $\phi_n \equiv \mathbb{P}\{\boldsymbol{\omega} = \omega_n\}$. The conditional distribution of cashflows is ordered according to MLRP. That is, $F(y|\boldsymbol{\omega} = \omega_i) \succ_{\text{MLRP}} F(y|\boldsymbol{\omega} = \omega_j)$ for all $i, j \in \{1, ..., N\}$ with $i > j$.[13] We further assume that $\mathbb{E}\{\boldsymbol{y}|\omega_N\} < \infty$. The issuer does not observe any information before trading with the liquidity supplier. Her prior beliefs about cashflows $\boldsymbol{y}$ are given by $\Psi \equiv \text{marg}_y F$.

Preferences. The liquidity supplier is risk-neutral. The valuation, in monetary terms, of a future cashflow y, for liquidity supplier ω_n is given by

$$u(y, \omega_n) \equiv \varphi(\omega_n)\, y + \nu(\omega_n).$$

where $\varphi(\omega_i), \nu(\omega_i)$ are both nonnegative and nondecreasing. The case where $\varphi(\omega) = 1$ and $\nu(\omega) = 0$ for all ω implies that heterogeneity only arises from different degrees of optimism about future cashflows (i.e., *belief heterogeneity*). The case where φ or ν are not constant corresponds to the case wherein higher types have a more productive technology that yields larger returns (i.e., *payoff heterogeneity*).

The expected utility of liquidity supplier ω_n from buying security $s(\cdot)$ at price p is then given by the following:

$$\begin{aligned} \mathbb{E}_n\{u_n(s)\} - p &= \int_{\mathbb{R}_+} u_n(s(y))\mathrm{d}F_n(y) - p \\ &= \int_{\mathbb{R}_+} (\varphi_n s(y) + \nu_n)\mathrm{d}F_n(y) - p \end{aligned}$$

where we write $u_n(s) = u(s, \omega_n)$, $\varphi_n = \varphi(\omega_n)$, $\nu_n = \nu(\omega_n)$, $F_n(y) = F(y|\omega_n)$, and $\mathbb{E}_n\{\cdot\} = \mathbb{E}\{\cdot|\omega_n\}$ for brevity. Henceforth, the absence of a subindex indicates the issuer's beliefs who, as explained above, does not observe additional information.

The issuer, on the other hand, is liquidity-constrained and discounts future cashflows with

burn cashflows and improve her payoff.

[13] This is equivalent to the conditional probability density function $f(y|\omega)$ satisfying log-supermodularity.

a factor $\delta \in [0, 1)$. For any contract specifying security s and price p, her payoff is given by

$$v(y, s, p) \equiv p + \delta(y - s(y)).$$

Assumption 1. *We assume that for any $s \in \mathcal{S}$, $\varphi_N \mathbb{E}_N \{s(\boldsymbol{y})\} > \delta \mathbb{E}\{s(\boldsymbol{y})\}$.*

Assumption 1 says that the highest type always values the securities offered by the issuer more than the latter. Intuitively, this assumption guarantees that the game admits positive gains from trade. In environments with pure belief heterogeneity, the assumption is trivially satisfied for example if the beliefs of the highest type dominate those of the issuer in the FOSD sense. In turn, in environments with pure payoff heterogeneity, the assumption requires that $\varphi_N > \delta$.

Mechanisms. Without loss of generality, we restrict attention to incentive-compatible, direct mechanisms. The issuer asks the liquidity supplier to report his type and offers and allocation and price given by $\mathcal{M} = \{s_n(y), p_n\}_{n=1}^N$, where s_n and p_n represent the security and the price offered to a liquidity supplier reporting ω_n, respectively. The issuer has all the bargaining power and designs $\mathcal{M}$.

Let $U_{\mathcal{M}}(\omega_i; \omega_j)$ represent the expected utility of a liquidity supplier whose true type is ω_j and reports ω_i under mechanism $\mathcal{M}$:

$$\begin{aligned} U_{\mathcal{M}}(\omega_i; \omega_j) &\equiv \int_0^\infty u_j(s_i(y))\, \mathrm{d}F_j(y) - p_i \\ &= \mathbb{E}_j \{u_j(s_i(\boldsymbol{y}))\} - p_i, \quad \forall i, j \in \{1, ..., N\}. \end{aligned}$$

Definition 1. We say that a mechanism $\mathcal{M}$ is *feasible* if it satisfies (i) individual rationality

$$[\mathrm{IR}_n]: \quad U_{\mathcal{M}}(\omega_n; \omega_n) \geq 0, \quad \forall n \in \{1, ..., N\},$$

and (ii) incentive compatibility

$$[\mathrm{IC}_{i,j}]: \quad U_{\mathcal{M}}(\omega_i; \omega_i) \geq U_{\mathcal{M}}(\omega_j; \omega_i), \quad \forall i, j \in \{1, ..., N\}.$$

The issuer designs a mechanism $\mathcal{M} = \{s_n^\star, p_n^\star\}_{n=1}^N$ that solves the following program

$$\begin{aligned} \max_{\{s_n, p_n\}_{n=1}^N} \mathbb{E}\{v(\boldsymbol{y}, s[\boldsymbol{\omega}], p(\boldsymbol{\omega}))\} &= \sum_{n=1}^N \phi_n \left(p_n + \delta \int_0^\infty (y - s_n(y))\, \mathrm{d}F_n(y) \right) \\ \text{s.t.} \quad & [\mathrm{IR}_i], [\mathrm{IC}_{i,j}], i, j \in \{1, ..., N\}. \end{aligned} \tag{1}$$

3 The Optimality of Debt Menus

In this section, we show that the optimal feasible mechanism consists of a *debt menu.* To prove this result, we first establish some basic properties that are instrumental to our analysis. We first prove that, regardless of the value of $\delta \in [0,1)$, all types $\omega > \omega_1$ enjoy strictly positive information rents thereby precluding the possibility full surplus-extraction mechanisms. We further show that, consistent with the standard screening problem, it is without loss of optimality to restrict attention to mechanisms that leave no information rents to the lowest type (no rents at the bottom) and that do not distort the efficient allocation of the highest type (no distortion at the top).

Proposition 1. *For any feasible mechanism $\mathcal{M} = \{s_n(\cdot), p_n\}_{n=1}^N$, the following properties are true:*

1. *For any $i, j \in \{1, ..., N\}$ with $i < j$, $U_{\mathcal{M}}(\omega_j; \omega_j) > U_{\mathcal{M}}(\omega_i; \omega_i) \geq 0$.*
2. *If $[IR_1]$ does not bind, then $\mathcal{M}$ is strictly dominated.*
3. *Suppose that assumption (1) holds. If $s_N(y) \neq y$ for all $y \in \mathbb{R}_+$, then $\mathcal{M}$ is strictly dominated.*

Property 1 in Proposition 1 says that in any feasible mechanism, all liquidity supplier types $\omega > \omega_1$ earn positive information rents. This result contrasts with the famous results by Crémer et al. [1987] and Crémer and McLean [1988] showing that when the issuer's and liquidity supplier's information signals are correlated, the issuer can capture all the surplus. In our case, $\boldsymbol{y}$ and $\boldsymbol{\omega}$ are correlated; yet the structure of the problem prevents the issuer from appropriating the whole surplus. The results follows from the assumptions that (a) the issuer is liquidity-constrained and (b) sells financial securities $s \in \mathcal{S}$. The assumption that the issuer need to raise cash in period 1 implies that the price p paid by the liquidity supplier cannot be made contingent on the future realizations of the asset's cashflows. In turn, the assumption that issuer sells financial securities ($s \in \mathcal{S}$) implies that they must exhibit cashflow monotonicity. This requirement imposes a constraint which precludes the design of the lotteries required for full-extraction. (Crémer and McLean [1988]).[14]

Interestingly, either (strict) belief or payoff heterogeneity on their own are enough to imply the results above. That is, even if the type ω is noninformative about cashflows or irrelevant

[14]Even if we dispense with the monotonicity assumption, the requirement that the securities must also satisfy limited liability would generally preclude full-extraction mechanisms (Crémer & Mclean [1985]) that typically require *deep pockets*.

for the liquidity supplier's preferences over cashflows, there will be positive information rents for all but the lowest type.

Properties 2 and 3 are standard. Property 2 follows from the fact that the lowest type, ω_1, obtains the lowest information rents (property 1). If this type obtains positive rents under a given mechanism, the issuer can increase all prices $\{p_n\}_n$ by the same amount until [IR$_1$] binds. Finally, property 3 obtains from the fact that the highest type, ω_N, values the asset more than the rest of the types and, by assumption (1), also more than the issuer. The issuer can therefore sell the whole asset to type ω_N and charge a price that only the type is willing to offer. Hence, it is optimal not to distort the allocation designed for the highest type.

3.1 Relaxing Incentive Constraints with Debt

The next property plays a special role for the subsequent results.

Definition 2. [Single crossing from below/above] We say that a function $h : \mathbb{R}_+ \to \mathbb{R}$ satisfies the property *single crossing from below* (SCFB) if there exists $y_0 \in \mathbb{R}_{++}$ so that $h(y) \leq 0$ for any $y < y_0$ and $h(y) \geq 0$ for any $y \geq y_0$. We say that h satisfies *strict single crossing from below* (SSCFB) if, in addition, the sets $\{y \in \mathbb{R}_+ : h(y) < 0\}$ and $\{y \in \mathbb{R}_+ : h(y) > 0\}$ have a positive (Lebesgue) measure. Similarly, we say that h satisfies *single crossing from above* (SCFA) or *strict single crossing from above* (SSCFA) if $-h$ satisfies SCFB or SSCFB, respectively.

Lemma 1. *Suppose that $(\boldsymbol{y}, \boldsymbol{\omega})$ satisfy the MLRP and that h satisfies the SCFB. If, for some $\omega' \in \Omega$, $\int_0^\infty h(y)\, dF(y|\omega') \geq 0$, then necessarily,*

$$\int_0^\infty h(y)\, \mathrm{d}F(y|\omega'') \geq 0, \ \forall \omega'' > \omega'.$$

If h satisfies $SSCFB$, then the second inequality is strict.

We now show that downward incentive constraints are maximally relaxed by assigning a debt security to the lowest type. Intuitively, among all securities that provide the same expected payoff (according to the beliefs of a specific liquidity supplier type), debt is the least preferred by higher types as it minimizes the payments for high realizations of $\boldsymbol{y}$ (i.e., the upside). Stated differently, debt securities minimize the information rents captured by high types.

Lemma 2. *Consider an arbitrary mechanism $\mathcal{M} = \{s_n, p_n\}_{n=1}^N$. Suppose that there exists $i \in \{1, ..., N\}$ such that $[IC_{j,i}]$ holds for all $j > i$. Then, the mechanism $\hat{\mathcal{M}} = \{\hat{s}_n, \hat{p}_n\}_{n=1}^N$ with*

$(\hat{s}_n, \hat{p}_n) = (s_n, p_n)$ *for all* $n \neq i$*, and* $(\hat{s}_i(y), \hat{p}_i) = (\min\{y, D_i\}, p_i)$*, with* D_i *defined such that* $\mathbb{E}_i (\min\{\boldsymbol{y}, D_i\} - s_i) = 0$*, satisfies* $[IC_{j,i}]$ *for all* $j > i$*. Moreover, whenever* $s_i \neq \hat{s}_i$*,* $[IC_{j,i}]$ *is slack for all* $j > i$*.*

Proof. Consider the mechanism $\hat{\mathcal{M}} = \{\hat{s}_n, \hat{p}_n\}_{n=1}^N$ described above. We show that the new mechanism relaxes incentive compatibility constraints. In fact, for any $j > i$, type ω_j's payoff from mimicking type ω_i decreases under $\hat{\mathcal{M}}$. To see this more clearly, observe that $s_i - \hat{s}_i$ satisfies SCFB in y. This fact, coupled with the log-supermodularity of $f(y|\omega)$ implies that the conditions in lemma 1 apply. Hence,

$$\mathbb{E}_i \{s_i - \hat{s}_i\} = 0 \Rightarrow \mathbb{E}_j \{s_i - \hat{s}_i\} \geq 0,$$

with strict inequality whenever s_i is not debt (i.e., $s_i \neq \hat{s}_i$ over a set with F-positive measure). This, in turn, implies that, for any $j > i$, type ω_j weakly prefers contract s_i over $\hat{s}_i$ as $\mathbb{E}_j \{u_j(s_i)\} > \mathbb{E}_j \{u_j(\hat{s}_i)\}$. Thus, the incentive compatibility constraints $[\text{IC}_{j,i}]$ are relaxed for all $j > i$ under $\hat{\mathcal{M}}$. □

Lemma 2 establishes that downward incentive constraints can be relaxed by allocating debt securities to low types. This does not necessarily imply that debt securities must be part of the optimal mechanism, as upward incentive constraints might be compromised by changing the securities for debt. In fact, as the steps leading to the proof of lemma 2 suggest, among all the securities s_i that provide type ω_i the same expected payoff $\mathbb{E}_i(s_i)$, the debt security $\hat{s}_i(y) = \min\{y, D_i\}$ is the one that provides maximal incentives to mimic for types $h < i$. The next section shows that despite this potential conflict, the issuer optimally sells debt securities to *all* types.

We note that for the lowest type, ω_1, for which there is no upward incentive constraints, lemma 2 directly implies that the optimal security must be debt.

Corollary 1. *Any feasible mechanism* $\mathcal{M} = \{s_n, p_n\}_{n=1}^N$ *for which there does not exist* $D > 0$ *such that* $s_1 = \min\{y, D\}$ *for (*F*-almost all)* y *(i.e.,* s_1 *is not a debt security), is strictly dominated by another feasible mechanism* $\hat{\mathcal{M}} = \{\hat{s}_n, \hat{p}_n\}_{n=1}^N$ *with* $\hat{s}_1(y) = \min\{y, D_1\}$ *for some* $D_1 > 0$*.*

3.2 Oriented Mechanisms

Before characterizing the optimal mechanism we identify a class of mechanisms that plays a special role in our environment.

Definition 3. We say that a feasible mechanism $\mathcal{M} = \{s_i, p_i\}_{i=1}^N$ is *oriented* if, for any $j, k \in \{1, ..., N\}$, with $k > j$, $\mathbb{E}_k \{s_k - s_j\} \geq 0$.

In words, a mechanism is oriented if any liquidity supplier type ω_k prefers his security to the ones tailored to lower types ω_j, with $k > j$. The concept of orientation is related to the monotonicity condition in the screening literature, but it neither implies nor is implied by the monotonicity of the allocation in the liquidity supplier's type.[15] Moreover, we show below that orientation does not follow from incentive compatibility (as in the single-dimensional case) but instead requires an optimality argument.

We argue below that we can restrict attention to the class of mechanisms satisfying orientation without loss of optimality.

3.3 Strong Liquidity Constraints

We consider first the case where $\delta = 0$. This captures the case where the issuer faces severe liquidity constraints and fully discounts the asset's future cashflows. The assumption allows us to highlight the main economic forces at play while maintaining tractability. We postpone the case $\delta \in (0, 1)$ to the next (sub)section and show that under mild conditions, the same results below still hold for the general case.

We use a replication argument to demonstrate that it is without loss of optimality to restrict attention to oriented mechanisms. To see this more clearly, consider an arbitrary feasible mechanism, $\mathcal{M} = \{s_n, p_n\}_{n=1}^N$, for which there exist j, k with $k > j$, $\mathbb{E}_k \{s_k - s_j\} < 0$. Then, we must have $p_j > p_k$ since otherwise the incentive constraint $[\text{IC}_{k,j}]$ would be violated. This, in turn, means that there exists an alternative mechanism, $\hat{\mathcal{M}}$, which is identical to $\mathcal{M}$ except for the fact that it offers contract $(\hat{s}_k, \hat{p}_k) = (s_j, p_j)$ to type ω_k. In other words, $\hat{\mathcal{M}}$ deletes the contract offered to type ω_k under mechanism $\mathcal{M}$ and replaces it with the contract offered to type ω_j. Note that $\mathbb{E}_k \{s_k - s_j\} < 0$, together with the assumption that $F(y|\omega_k) \succ_{\text{MLRP}} F(y|\omega_j)$ and the monotonicity of $u(x, \cdot)$ in ω, jointly imply that

$$U_{\hat{\mathcal{M}}}(\omega_k; \omega_k) = \mathbb{E}_k \{u_k(s_j)\} - p_j > \mathbb{E}_j \{u_j(s_j)\} - p_j \geq 0.$$

Therefore, the new mechanism satisfies $[\text{IR}_k]$. Furthermore, all other incentive constraints remain uncompromised under the new mechanism as the contract (s_j, p_j) was already available under $\mathcal{M}$. The new mechanism $\hat{\mathcal{M}}$ thus strictly raises more funds than $\mathcal{M}$. When $\delta = 0$, the

[15] When the allocation space is single-dimensional (e.g., the probability of selling the good), this property is equivalent to the standard monotonicity condition, implied by incentive constraints. The corresponding analog in the infinite-dimensional case can be appreciated in inequality (25) in the Appendix.

issuer is maximally impatient and strictly prefers the mechanism $\hat{\mathcal{M}}$ over $\mathcal{M}$ as it raises more funds. We note that in contrast to the standard argument where incentive compatibility alone implies monotonicity, in the current environment, orientation is a consequence of both incentive compatibility and optimality.

Lemma 3. [Oriented Mechanisms] *Any feasible mechanism $\mathcal{M} = \{s_n, p_n\}_{n=1}^N$ that is not oriented is strictly dominated by another feasible, oriented mechanism.*

Equipped with the last result, we are now ready to present the main result of this subsection.

Theorem 1. *There exists a unique optimal mechanism, $\mathcal{M}^\star = \{s_n^\star, p_n^\star\}_{n=1}^N$, satisfying the following properties. For any $n \leq N-1$, there exists $D_n > 0$ so that $s_n^\star(y) = \min\{y, D_n\}$ and $s_N^\star(y) = y$, for all y.*

We provide a sketch of the proof below, which is divided into 3 steps.

In Step 1, we show that for any oriented mechanism, when security s_i is debt, then for any pair of types above i, $k > j > i$, the downward incentive constraints [$\text{IC}_{k,j}$] and [$\text{IC}_{j,i}$] imply the *global* downward constraint [$\text{IC}_{k,i}$]. The global downward incentive constraints are therefore redundant.

The intuition behind Step 1 is as follows. When s_i is a debt contract, then for any j, k with $k > j > i$, if type ω_j prefers his security s_j over s_i, then type ω_k must also prefer s_j over s_i. The standard argument used in the screening literature follows from the monotonicity of the allocation rule (which is implied by incentive compatibility) and the supermodularity of the liquidity supplier's payoff in both his type and the allocation. In the current setting, it is not clear what the correct notion should be of the supermodularity of the liquidity supplier's payoff in his type and the security, an infinite-dimensional object. Instead, we leverage the fact that when s_i is a debt contract, s_i crosses other securities from above. Lemma 1 then implies that for any s_j with $\mathbb{E}_j\{s_j - s_i\} \geq (>)\, 0$ (which is always true for oriented mechanisms), it must be the case that $\mathbb{E}_k\{s_j - s_i\} \geq (>)\, 0$. This in turn means that if type ω_j, according to his beliefs, does not mimic type ω_i, then neither does type ω_k.

In Step 2, we use the result obtained in Step 1 to show that when s_i is debt, then the local downward incentive constraint [$\text{IC}_{i+1,i}$] must bind. The argument is obtained by contradiction. Suppose that for some $i \in \{1, ..., N-1\}$, [$\text{IC}_{i+1,i}$] is satisfied with slackness; then the argument in Step 1 implies that for any $k \geq i+1$, the incentive constraints [$\text{IC}_{k,i}$] are slack. The issuer can therefore increase the transfers p_k for all types $k \geq i+1$ without spoiling incentive compatibility. We conclude that at any undominated mechanism, constraint [$\text{IC}_{i+1,i}$] must bind.

Finally, in Step 3, we show that for any oriented mechanism $\mathcal{M}$, if we let $i+1 \leq N$ be the smallest type for whom his security s_{i+1} is not debt, we can improve upon the issuer's payoff by swapping security s_{i+1} for the payoff-equivalent debt contract s_{i+1}^D (according to ω_{i+1}'s beliefs), without spoiling the upward incentive constraints, [$\text{IC}_{h,i+1}$], for $h \leq i$. Here we sketch the argument for the case $h = i$. Let s_{i+1}^D be a debt security with $\mathbb{E}_{i+1}\left\{s_{i+1}^D - s_{i+1}\right\} = 0$, the issuer can relax the downward incentive constraints without spoiling the upward incentive constraints. Indeed, we observe that

$$\begin{aligned} p_{i+1} - p_i &= \mathbb{E}_{i+1}\left\{u_{i+1}\left(s_{i+1}\right) - u_{i+1}\left(s_i\right)\right\} \\ &= \mathbb{E}_{i+1}\left\{u_{i+1}\left(s_{i+1}^D\right) - u_{i+1}\left(s_i\right)\right\} \\ &\geq \mathbb{E}_{i}\left\{u_{i+1}\left(s_{i+1}^D\right) - u_{i+1}\left(s_i\right)\right\}, \end{aligned} \tag{2}$$

where the first equality follows from the result in Step 2, and the second equality is obtained by the construction of s_{i+1}^D. The inequality, in turn, follows from noting that the mechanism $\mathcal{M}$ is oriented and therefore

$$\mathbb{E}_{i+1}\left\{s_{i+1}^D\right\} = \mathbb{E}_{i+1}\left\{s_{i+1}\right\} \geq \mathbb{E}_{i+1}\left\{s_i = \min\left\{\boldsymbol{y}, D_i\right\}\right\}.$$

This means that $s_{i+1}^D - s_i$ is nondecreasing, which, coupled with the MLRP assumption and the monotonicity of $u\left(s,\omega\right)$ in ω, implies the result.

Inequality (2) implies that type ω_i does not mimic type ω_{i+1} when the latter's security is replaced by its payoff-equivalent debt security; hence, the upward incentive constraint [$\text{IC}_{i,i+1}$] is satisfied. We show in the Appendix that a similar argument can be made to show that for any $h < i$, [$\text{IC}_{h,i+1}$] is also satisfied in the new mechanism.

Theorem 1 shows that to maximize the funds raised from an informed investor, restricting attention to debt menus does not result in loss of optimality. This characterization, together with the result in Lemma 3, jointly imply that the face values associated with the optimal debt securities must be monotone in the liquidity supplier's type. Lemma 4 below crystallizes this point and shows that the issuer's optimal mechanism is the debt menu that maximizes her virtual valuation across all debt menus with monotone face values. Our characterization thus implies that, similar to the classical screening problem (e.g., Mussa and Rosen [1978]), the allocation space is reduced to a single-dimensional variable, namely, the face value of the debt securities and a simple monotonicity restriction.

Interestingly, theorem 1 further implies that exploiting the possibility of introducing securities with different degrees of information sensitivity in the menu is *ineffective* at screening the liquidity supplier's private information. In subsection 3.5, we substantiate this claim by formalizing the intuition that the liquidity supplier's information rents are monotone in the securities' information sensitivity.

3.3.1 Characterization of the Optimal Mechanism

Equipped with the result in theorem 1, we proceed to characterize the optimal mechanism with securities. The previous results imply that we can restrict attention to menus of debt securities, that is, mechanisms satisfying that, for any n, $s_n = \min\{y, D_n\}$ for all y, where $D_n > 0$ and is increasing in n, and $D_N = +\infty$. Moreover, Step 2 in the proof theorem 1 implies that, without loss, we can restrict attention to mechanisms satisfying that, for any $j > 1$, $[\mathrm{IC}_{j,j-1}]$ binds and, because of proposition 1, further satisfying $p_1 = \mathbb{E}\{\min\{\boldsymbol{y}, D_1\}\}$. Together, these properties imply the following characterization.

Lemma 4. *The issuer's problem can be reformulated as*

$$\begin{aligned} \max_{\{D_n\}_{n=1}^N} \quad & \sum_{n=1}^N \phi_n \int_0^\infty u_n(\min\{y, D_n\}) \left(1 - \left(\frac{1-\Phi_n}{\phi_n}\right)\left(\frac{f_{n+1}(y) - f_n(y)}{f_n(y)}\right)\right) \mathrm{d}F_n(y) \quad (3) \\ \text{s.t.} \quad & D_n \text{ nondecreasing in } n. \end{aligned}$$

3.4 General Case

We now consider the general case where $\delta \in [0, 1)$. The results below readily extend to the case where Ω is continuous (i.e., $\Omega = [\underline{\omega}, \bar{\omega}] \subseteq \mathbb{R}_+$). We maintain the notation of the discrete case below for comparability with the former section. In the Appendix we provide a general statement and proof to encompass the continuum case. To simplify the exposition, we focus hereafter on the case of pure heterogeneity in beliefs. That is, $u_n(s) = s$ for all n (i.e., $\varphi_n = 1, \nu_n = 0$). We make two standard assumptions.

Assumption 2. *The inverse hazard rate $\frac{1-\Phi(\omega)}{\phi(\omega)}$ is nonincreasing in ω.*

Assumption 3. *The conditional distribution function $F(y|\omega)$ is convex in ω.*

Assumption 2 is standard in the mechanism design literature (see, e.g., Myerson [1981]). Intuitively, this assumption curbs the information rents that need to be forgone to higher types. Assumption 3, on the other hand, requires the differential

$$\Delta_n(y) \equiv 1 - F_{n+1}(y) - (1 - F_n(y))$$

to decrease as n increases. This effect is reminiscent to the requirement that the marginal value of increasing the buyer's allocation to be concave in his type (see, e.g., Fudenberg and Tirole [1991]). When the issuer restricts herself to sell debt securities to all types (which, as we prove below, is a fortiori optimal), the allocation designed for type ω_n is determined by the face value of his debt security, D_n. The marginal value of increasing type ω_n's allocation is therefore given by

$$\begin{aligned}\frac{\partial}{\partial D_n}\int_{\mathbb{R}_+}\min\{y,D_n\}\,\mathrm{d}F(y|\omega_n) &= \frac{\partial}{\partial D_n}\left(\int_0^{D_n} y\mathrm{d}F_n(y)+D_n(1-F_n(D_n))\right)\\ &= 1-F_n(D_n).\end{aligned}$$

Thus, assumption 3 implies that the marginal value of increasing the liquidity supplier's allocation is concave in ω_n. Our next result shows that under these two assumptions, the issuer's optimal mechanism consists of a debt menu. Furthermore, the optimal menu does not feature pooling.

Theorem 2. *Suppose that assumptions (1), (2), and (3) hold. Then, the optimal mechanism* $\mathcal{M}^\star=\{s_n^\star,p_n^\star\}_{n=1}^N$ *consists of a debt menu. For every* $n\in\{1,...,N\}$, $s_n^\star=\min\{y,D_n^\star\}$, *where* $D_n^\star$ *is strictly monotone in* n *and is implicitly characterized by the unique solution to*[16]

$$\int_{D_n}^\infty\left(1-\delta-\left(\frac{1-\Phi_n}{\phi_n}\right)\left(\frac{f_{n+1}(y)-f_n(y)}{f_n(y)}\right)\right)dF_n(y)=0. \tag{5}$$

We provide a sketch the proof. Consider the relaxation of the issuer's problem (program 1) where only the local incentive constraints are required. That is, the issuer solves

$$\begin{aligned}\max_{\{s_n,p_n\}_{n=1}^N}\quad & \sum_{n=1}^N\phi_n\left(p_n+\delta\int_0^\infty(y-s_n(y))\,\mathrm{d}F_n(y)\right)\\ \text{s.t.}\quad & [\mathrm{IR}_i],[\mathrm{IC}_{i,j}],i\in\{1,...,N\},j\in\{i-1,i+1\}.\end{aligned} \tag{6}$$

Denote by $\hat{\mathcal{M}}=\{\hat{s}_n,\hat{p}_n\}_{n=1}^N$ the solution to the program above. Clearly, the issuer's payoff under $\hat{\mathcal{M}}$ is weakly higher than under the mechanism $\mathcal{M}^\star$ that solves program (1) as the set of constraints is a strict subset of the latter. First, we argue that in the relaxed problem,

[16]When Ω is continuous, we further require that $\mathbb{E}\left\{\boldsymbol{y}\frac{\frac{\partial f(\boldsymbol{y}|\omega)}{\partial\omega}}{f(\boldsymbol{y}|\omega)}|\omega\right\}<\infty$. In that case, $D^\star(\omega)$ is implicitly characterized by

$$\int_{D^\star(\omega)}^\infty\left\{1-\delta-\frac{1-\Phi(\omega)}{\phi(\omega)}\frac{\frac{\partial}{\partial\omega}f(y|\omega)}{f(y|\omega)}\right\}dF(y|\omega) \quad=\quad 0. \tag{4}$$

local-downward incentive constraints must bind. Otherwise, if some constraint $[\mathrm{IC}_{i+i,i}]$ does not bind, the issuer can raise the prices to all types above ω_i and still satisfy the feasibility constraints. We can therefore rewrite the issuer's payoff as

$$\sum_{n=1}^{N}\phi_n\int_0^{\infty}s_n\left(y\right)\left(1-\delta-\left(\frac{1-\Phi_n}{\phi_n}\right)\left(\frac{f_{n+1}\left(y\right)-f_n\left(y\right)}{f_n\left(y\right)}\right)\right)\mathrm{d}F_n\left(y\right).$$

We argue in the Appendix that the expression above is pointwise maximized by choosing, for each $n\in\{1,...,N\}$, a debt security with face value $D_n^{\star}$, as defined in (5). Under assumptions (2) and (3), the sequence $\{D_n^{\star}\}_{n=1}^{N}$ is monotone. This implies that the set of debt securities $\{\hat{s}_n=\min\{y,D_n^{\star}\}\}_{n=1}^{N}$ is oriented. The orientation of $\{\hat{s}_n\}_{n=1}^{N}$ in turn implies that we can use the same arguments establishing Step 1 in the proof of Theorem 1 to show that any oriented mechanism respecting local, downward incentive constraints is globally incentive compatible (i.e., the incentive constraints $[\mathrm{IC}_{i,j}], i\in\{1,...,N\}$, $|j-i|>1$ are also satisfied). Thus, the solution to the relaxed program $\hat{\mathcal{M}}$ is also feasible at the original program (1) and must therefore be optimal.

3.5 Information Sensitivity and Information Rents

Definition 4. We sat that security $s''\in\mathcal{S}$ is more information sensitive than $s'\in\mathcal{S}$, if $s''-s'$ has the SSCFB property.

Intuitively, more information sensitive securities increase the exposure of the security owner to the asset's cashflow realizations. That $s''-s'$ has the SSCFB property implies that for low cash flow realizations $s''(y)<s'(y)$ the security owner has a smaller payoff under s'' than under s', whereas the opposite holds true, for high cashflow realizations.

We argue that the liquidity supplier's information rents are closely related to the securities' *information sensitivity*. Indeed, the steps leading to the result in Theorem 4, imply that, for any feasible menu of securities $\{s[\omega]\}_{\omega\in\Omega}$, the issuer's objective can be expressed as

$$\int_{\Omega}\left(\underbrace{(1-\delta)\mathbb{E}\left\{s\left(\boldsymbol{y}|\omega\right)|\omega\right\}}_{\text{Gains from trade}}-\underbrace{\left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right)\mathbb{E}\left\{s\left(\boldsymbol{y}|\omega\right)\left(\frac{\frac{\partial}{\partial\omega}f(\boldsymbol{y}|\omega)}{f(\boldsymbol{y}|\omega)}\right)|\omega\right\}}_{\text{Informational Rents}}\right)\mathrm{d}\Phi\left(\omega\right).$$

For any $\omega \in \Omega$, and any feasible mechanism $\mathcal{M} = \{s[\omega], p(\omega)\}_{\omega\in\Omega}$, let

$$I^{\mathcal{M}}(\omega) \equiv \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\mathbb{E}\left\{s(\boldsymbol{y}|\omega)\left(\frac{\frac{\partial}{\partial\omega}f(\boldsymbol{y}|\omega)}{f(\boldsymbol{y}|\omega)}\right)|\boldsymbol{\omega}=\omega\right\}$$

represent type ω's *information rents* under mechanism $\mathcal{M}$. This expression represents the amount of rents that need to be paid to every type above ω to not pretend to be type ω. When the liquidity supplier's type is commonly known, the issuer maximizes the expected value of the security sold to the liquidity supplier. In that case, the issuer optimally sells the whole asset to the liquidity supplier, who is the efficient holder of the asset, and extracts all rents from him. Instead, when the liquidity supplier has private information, the issuer needs to leave information rents to the liquidity supplier, which entails distorting allocative efficiency. As theorem 1 dictates, the issuer is better off by holding on to the high realizations of the asset's cashflows despite the fact that she assigns a smaller value to them. Doing so allows her to minimize the information rents she leaves to the highest types.

To elaborate further, consider the standard screening problem, wherein a buyer purchases an asset from an issuer constrained to sell pure equity (i.e., the whole asset). In that environment, it is without loss to restrict attention to direct mechanisms specifying, for each type ω, the probability of trading, $\alpha(\omega) \in [0,1]$, and the transfers $p(\omega) \geq 0$. That environment is equivalent to an issuer restricted to selling equity stakes (e.g., stocks), i.e., $\left\{s^E(y|\omega) = \alpha(\omega)\cdot y\right\}_{\omega\in\Omega}$, at prices $\{p(\omega)\}_{\omega\in\Omega}$.[17]

Selling information sensitive securities (such as these equity stakes), however, leaves large information rents to the liquidity supplier. The issuer can minimize these rents by reducing the information sensitivity of the securities in her menu. Indeed, consider any feasible mechanism $\mathcal{M}^E = \left\{s^E[\omega], p^E(\omega)\right\}_{\omega\in\Omega}$, with $s^E(y|\omega) = \alpha(\omega)\cdot y$, for all ω. Construct an alternative mechanism with information-insensitive securities, $\mathcal{M}^{II} = \left\{s^{II}[\omega], p^{II}(\omega)\right\}_{\omega\in\Omega}$, where for each ω, $\mathbb{E}\left\{s^{II}(\boldsymbol{y}|\omega)|\omega\right\} = \mathbb{E}\left\{s^{E}(\boldsymbol{y}|\omega)|\omega\right\}$, and $s^{II}[\omega]$ is *less information sensitive* than $s^E[\omega]$; that is, $s^{II}[\omega] - s^E[\omega]$ is SSCFA. Then,

$$\begin{aligned}
I^{\mathcal{M}^{II}}(\omega) - I^{\mathcal{M}^{E}}(\omega) &= \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\int_0^\infty \left(s^{II}(y|\omega) - s^E(y|\omega)\right)\frac{\partial}{\partial\omega}f(y|\omega)\mathrm{d}y \\
&= \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\int_0^\infty \left(s^{II}(y|\omega) - s^E(y|\omega)\right)\lim_{\delta\downarrow 0}\left(\frac{f(y|\omega+\delta)-f(y|\omega)}{\delta}\right)\mathrm{d}y \\
&< 0,\ \forall\omega\in\Omega
\end{aligned}$$

[17]The proof of Example 1 contains the derivation of the optimal mechanism restricted to this class of securities.

where the inequality follows from the construction of $\left\{s^{II}[\omega]\right\}_{\omega\in\Omega}$ and the inequality is a direct implication of Lemma 1 which jointly imply that, for any $\delta > 0$,

$$\begin{aligned}\int_0^\infty \left(s^{II}(y|\omega) - s^E(y|\omega)\right)\left(\frac{f(y|\omega+\delta) - f(y|\omega)}{\delta}\right)\mathrm{d}y &= \int_0^\infty \left(s^{II}(y|\omega) - s^E(y|\omega)\right)\frac{f(y|\omega+\delta)}{\delta}\mathrm{d}y \\ &< 0.\end{aligned}$$

We conclude that, regardless of the liquidity supplier's private signal, information rents are strictly smaller when the issuer offers more informational-insensitive securities. The fact that these latter securities are constructed keeping the liquidity supplier's valuation unchanged, then implies that the issuer strictly benefits form her ability to design securities which are less sensitive to the liquidity supplier's information. This prediction stands in sharp contrast with the typical finding in the security design literature according to which, when trading, the informed agent obtains information-sensitive securities.[18]

4 Information and Monotone Comparative Statics

Motivated by the important role of the liquidity supplier's private information in determining the optimal mechanism, we explore how changes in the quality of the liquidity supplier's private signal affects the issuer's optimal mechanism and her ability to raise funds. In practice, the quality of the liquidity supplier's information is influenced by different channels. It may vary, for example, because of the introduction of new regulation. In the context of asset backed securities (ABS), under Regulation AB, the SEC imposes disclosure requirements for asset-backed securities offerings. [19] The information provided by the issuer then allows the liquidity provider to assess the value of the security according to his idiosyncratic preferences and beliefs. The quality of the liquidity supplier's information might also evolve with technological changes to the market participants' data processing tools, and through the liquidity supplier's due diligence efforts. We remain agnostic about the determinants of the underlying information structure and, instead, explore how changes to the quality of the investors' information affect the issuer's funding capacity.

To simplify the derivations, we focus henceforth on the case with a continuum of types mentioned above and formally derived in Theorem 4 in the Appendix. We show below that as the liquidity supplier's private signal becomes more *accurate* (Lehmann [1988]), the issuer optimally offers debt instruments with smaller face values. Perhaps surprisingly, however, the

[18] See, e.g., Gorton and Pennacchi [1990], Nachman and Noe [1994], DeMarzo and Duffie [1999], Biais and Mariotti [2005].

[19] Please refer to https://www.sec.gov/corpfin/divisionscorpfinguidanceregulation-ab-interpshtm.

geometry of the optimal securities, i.e., the concavity of debt on the asset's future cashflows, implies that improving the accuracy of the liquidity supplier's information increases his valuation for these securities. This effect is similar to a reduction in uncertainty for a risk-averse agent, which increases his utility. The overall effect on both surplus and the agents' payoffs is thus ambiguous.

Below, we provide an example that showcases the novel effect described above and challenges some economic intuitions from the classical screening problem.

Example 1. Consider the case with $\delta = 0$ (strong liquidity constraints). Suppose that $\boldsymbol{\omega} \sim U[0,1]$ and that for $\theta \in [1/3, 1]$, $\boldsymbol{y}^\theta$ is constructed as follows. With probability θ, $\boldsymbol{y}^\theta = \boldsymbol{\omega}$, whereas with probability $1-\theta$, $\boldsymbol{y}^\theta \sim U[0,1]$, independent of $\boldsymbol{\omega}$. That is,

$$F^\theta(y|\omega) = \Pr\left\{\boldsymbol{y}^\theta \leq y | \boldsymbol{\omega} = \omega\right\} = \theta \cdot 1\{\omega \leq y\} + (1-\theta)\, y.$$

The following properties are true:

(a) For any $\theta'', \theta' \in (0,1)$, $\theta'' > \theta'$, $F^{\theta''}$ is more accurate (Lehmann [1988]) than $F^{\theta'}$.[20]

(b) Suppose the issuer is restricted to use linear securities, i.e., for all $\omega \in [0,1]$,

$$s[\omega] \in \mathcal{S}^E \equiv \{s \in \mathcal{S} : \exists \alpha > 0, s(y) = \alpha y, \forall y \in [0,1]\}.$$

Then, the (restricted) optimal mechanism is characterized by

$$\alpha_\theta^E(\omega) = 1\left\{\omega \geq \omega_\theta^E \equiv \frac{3\theta - 1}{4\theta}\right\}.$$

Furthermore, under this mechanism $\mathbb{E}\left\{p_\theta^E(\boldsymbol{\omega})\right\} = \frac{(1+\theta)^2}{16\theta}$.

(c) The issuer's (unrestricted) optimal mechanism is a debt menu $\{\min\{y, D_\theta^*(\omega)\}\}_{\omega \in [0,1]}$ with face values characterized by

$$D_\theta^*(\omega) = \omega,\ \forall \omega \in [0,1], \forall \theta \in (0,1).$$

Under this mechanism, information rents are 0 for all $\omega \in [0,1]$; Furthermore, $\mathbb{E}\{p_\theta^*(\boldsymbol{\omega})\} = \mathbb{E}\{\mathbb{E}\{\min(\boldsymbol{y}, \boldsymbol{\omega})|\boldsymbol{\omega}\}\} = \frac{2+\theta}{6}$.

Example 1 underscores some fundamental differences with the case where the issuer is constrained to use linear instruments, a typical assumption in the screening literature. Under the restriction, the optimal mechanism consists of a posted price. The issuer sells the whole

[20] The formal definition is provided in the next subsection.

asset to all liquidity supplier types above the critical type ω_θ^E at price $\mathbb{E}\left\{\boldsymbol{y}|\omega_\theta^E\right\}$, leaving strictly positive information rents to all types strictly above the critical type. Furthermore, as the accuracy of the liquidity supplier's private information increases, the issuer increases the posted price destroying both surplus and the expected proceeds from the security sale (indeed, $\mathbb{E}\left\{p_\theta^E(\boldsymbol{\omega})\right\}$ decreases with θ).

In contrast, when the issuer can flexibly design the security, she optimally sells debt instruments to all liquidity supplier types. Interestingly, as we prove in the Appendix, under the optimal mechanism, the issuer leaves *no information rents*.[21] Using a debt menu allows the issuer to alleviate incentive compatibility issues and extract all the surplus generated by these securities. Strikingly, as the liquidity supplier's private information becomes more accurate, the issuer *increases* expected proceeds from the sale. As we formally prove below, this counterintuitive result is a direct consequence of the geometry of the optimal securities. The concavity of debt implies that the liquidity suppliers valuation for these securities increases when she is faced with a reduction in uncertainty.

The example above is special in some dimensions. First, the optimal mechanism is invariant in θ. As we prove below in Theorem 3, the set of optimal face values is nonincreasing in the accuracy of the liquidity supplier's private signal. This provides a countervailing effect that reduces the issuer's expected revenue. Second, in the example, the issuer is able to extract all the rents with debt securities. This property need not extend to more general environments. We show that the information rents have the single crossing property in the accuracy of the liquidity supplier's signal. This provides another countervailing effect that reduces the issuer's revenue. In Example 1, neither of the two effects are present, which leads to the surprising result that the issuer benefits and raises more funds when she faces a more informed liquidity supplier.

4.1 Information Accuracy [Lehmann [1988]]

Suppose that conditional on the liquidity supplier drawing a private type $\boldsymbol{\omega} = \omega$, the distribution of the asset's future cashflows $\boldsymbol{y}$ is drawn according to the kernel function $F(y|\omega) = \Pr\{\boldsymbol{y} \leq y|\boldsymbol{\omega} = \omega\}$. Following the tradition in the information literature, we refer to $F(y|\omega)$ as an *experiment*. The experiment F and the prior distribution of $\boldsymbol{\omega}$, Φ, uniquely determine the joint distribution

$$\mathbf{F}_\Phi(y, \omega) \equiv \int_{\Omega\times\mathbb{R}_+} F(y|\tilde{\omega})\, 1\{\tilde{\omega} \leq \omega\}\, \Phi(\mathrm{d}\tilde{\omega}).$$

[21] This result contradicts property 1 in Proposition 1. The assumption that the example violates is the requirement on F^θ to not have atoms in y.

Throughout the analysis, we maintain the assumption that the distribution of $\mathbf{F}_\Phi(y,\omega)$ admits a density $\mathbf{f}_\Phi(y,\omega)$, which satisfies MLRP. We refer to the induced marginal distribution of the asset's cashflows as $\Psi(F,\Phi) = \text{marg}_{\boldsymbol{y}}\mathbf{F}_\Phi \in \Delta\mathbb{R}_+$.

Below, we formally introduce a natural ordering to rank the amount of information embedded in the liquidity supplier's private type, $\boldsymbol{\omega}$, about the asset cashflows, $\boldsymbol{y}$.

Definition 5. [Lehmann [1988]] Consider two random variables $\boldsymbol{y}''$ and $\boldsymbol{y}'$ representing the asset's future cashflows, and let $F''(y|\omega)$ and $F'(y|\omega)$ be the experiments associated with them, respectively. We say that F'' is *more accurate* about $\boldsymbol{\omega}$ than F', which we write as $F'' \succeq_{\text{Lehmann}} F'$ if for any y,[22]

$$F''^{-1}(F'(y|\omega)|\omega) \text{ is nondecreasing in } \omega.$$

Accuracy has been used in the information economics literature to perform monotone comparative statics in the context of auctions (Persico [2000] and Ganuza and Penalva [2010]), moral hazard (Kim [1995], Jewitt [2007]), and career concerns (Dewatripont et al. [1999]).

4.2 Accuracy, Supermodularity, and Informativeness

We argue that accuracy (Lehmann [1988]) is an appealing notion of informativeness in our environment for three reasons. First, provided that signals satisfy MLRP, accuracy is less restrictive than the Blackwell ordering in that it compares more signal structures.[23] Furthermore, as the next proposition shows, accuracy implies the standard notion of informativeness usually assumed in the information design literature. Second, the concept of accuracy is tightly related to the idea of interdependence. When an experiment is more accurate, the comovement between fundamentals and signals becomes stronger. Finally, the Lehmann order compares experiments (i.e., conditional distributions) as opposed to comparing joint distributions. Intuitively, we want to change the quality of the liquidity supplier's information without changing the distribution of liquidity supplier types Φ or the distribution of the asset's future cashflows Ψ. This is natural since changing the marginal Φ changes the relative likelihood of facing different liquidity supplier types, which directly changes the amount of information

[22] An alternative and perhaps more fundamental definition states that any decision-maker with a (Bernoulli) utility function supermodular in the action and the underlying variable ω would prefer the information obtained by learning $\boldsymbol{y}''$ over the information obtained from $\boldsymbol{y}'$. See Jewitt [2007] and Quah and Strulovici [2009] for a detailed discussion.

[23] Indeed, provided that signals satisfy the MLRP, any two signals ordered according to Blackwell are also ordered according to Lehmann [1988]. When the state space is binary, both notions of informativeness coincide (see, e.g., Jewitt [2007]). More recently, Kim [2022] showed that Lehmann domination is closely related to the concept of quasigarbling, a generalization of information garbling.

rents that must be provided to the different liquidity supplier types. In turn, changing the marginal Ψ changes the quality of the underlying asset and hence the liquidity supplier's valuation for the securities offered. In what follows, then, we fix a given marginal distribution, Φ, and we study the effect of increasing the accuracy of experiment F, while keeping the marginal distribution of cashflows $\Psi(F, \Phi) = \text{marg}_{\boldsymbol{y}} \mathbf{F}_{\Phi}$ unchanged. The results described in our next proposition guarantee that as we increase the accuracy of the experiment F, we do not change the primitives of the issuer's problem beyond the effect induced via the accuracy of the liquidity supplier's information making the comparative statics exercise coherent.

Our next result summarizes the appeal of using accuracy as the appropriate ordering to compare the informativeness of different signals structures.

Proposition 2. *Consider an arbitrary marginal distribution $\Phi \in \Delta\Omega$, and suppose that $F'' \succeq_{Lehmann} F'$. Let $\mathbf{F}''_{\Phi}$ and $\mathbf{F}'_{\Phi}$ be the induced joint distributions and assume that $\Psi = marg_{\boldsymbol{y}} \mathbf{F}''_{\Phi} = marg_{\boldsymbol{y}} \mathbf{F}'_{\Phi}$. Then,*

(i) For any supermodular function $v(\omega, y)$,

$$\int_{\mathbb{R}_+ \times \Omega} v(y, \omega) \, \mathrm{d}\mathbf{F}''_{\Phi}(y, \omega) \geq \int_{\mathbb{R}_+ \times \Omega} v(y, \omega) \, \mathrm{d}\mathbf{F}'_{\Phi}(y, \omega).$$

In other words, $\mathbf{F}''_{\Phi}$ dominates $\mathbf{F}'_{\Phi}$ in the supermodular order, $\mathbf{F}''_{\Phi} \succeq_{spm} \mathbf{F}'_{\Phi}$.

(ii) $Cov_{\mathbf{F}''_{\Phi}}(\boldsymbol{\omega}, \boldsymbol{y}) \geq Cov_{\mathbf{F}'_{\Phi}}(\boldsymbol{\omega}, \boldsymbol{y})$.

(iii) Let $\boldsymbol{z}'' \equiv \mathbb{E}_{\mathbf{F}''_{\Phi}}(\boldsymbol{y}|\boldsymbol{\omega})$ and $\boldsymbol{z}' \equiv \mathbb{E}_{\mathbf{F}'_{\Phi}}(\boldsymbol{y}|\boldsymbol{\omega})$, and denote by H'' and H' the respective cumulative distribution functions of $\boldsymbol{z}''$ and $\boldsymbol{z}'$, respectively. Then, for any convex function $\gamma : \mathbb{R}_+ \to \mathbb{R}$,

$$\int \gamma(z) \, \mathrm{d}H''(z) \geq \int \gamma(z) \, \mathrm{d}H'(z).$$

In other words, $\boldsymbol{z}''$ dominates $\boldsymbol{z}'$ in the convex order, $\mathbb{E}_{\mathbf{F}''_{\Phi}}(\boldsymbol{y}|\boldsymbol{\omega}) \succeq_{cvx} \mathbb{E}_{\mathbf{F}'_{\Phi}}(\boldsymbol{y}|\boldsymbol{\omega})$.

Proposition 2 shows the appeal of using accuracy to rank the informativeness of different experiments. First, as an intermediate step, we recall that the Lehmann ordering, which compares experiments, is tightly related to the supermodular order, which in turn compares joint distributions. As explained by Meyer and Strulovici [2012], the fact that $\mathbf{F}''_{\Phi} \succeq_{\text{spm}} \mathbf{F}'_{\Phi}$ implies that the degree of interdependence of $(\boldsymbol{y}, \boldsymbol{\omega})$ is larger under $\mathbf{F}''_{\Phi}$ than under $\mathbf{F}'_{\Phi}$. This means that we can interpret increments in accuracy as changes in the joint distribution of $(\boldsymbol{y}, \boldsymbol{\omega})$, which increase their degree of correlation while keeping their marginal distributions constant. Furthermore, as we show below, this property allows us to compare how the liquidity supplier's valuation for debt securities changes as we increase the accuracy of his private signal.

Finally, Proposition 2 also shows that the concept of accuracy is closely related to the classical notion of informativeness in the information economics literature. Claim (iii) shows that when F'' is more accurate than F', then the random variable capturing the posterior estimates induced by learning the realization of $\boldsymbol{\omega}$ under F'', $\mathbb{E}_{\mathbf{F}''_{\Phi}}(\boldsymbol{y}|\boldsymbol{\omega})$ is a mean-preserving spread of the analogous random variable capturing the posterior estimates under F', $\mathbb{E}_{\mathbf{F}'_{\Phi}}(\boldsymbol{y}|\boldsymbol{\omega})$. In other words, when the accuracy of the experiment improves, the liquidity supplier's private information becomes more informative in the classical sense about the asset's future cashflows.

4.3 Information and the Issuer's Funding Capacity

In this section, we leverage the geometry of the optimal mechanism, i.e., the remarkable feature that it consists of a menu of debt securities, to show that as the liquidity supplier's private information becomes more accurate, (i) the liquidity supplier's valuation for any debt security increases, and (ii) the issuer sells *smaller* securities, i.e., debt securities with smaller face values. To the best of our knowledge, this is the first paper to perform monotone comparative statics using information orderings in the context of security design.

Our first result formalizes the novel effect described above, according to which increasing the accuracy of the experiment, F, implies that from the point of view of the liquidity supplier, the distribution of cashflows becomes less risky. The concavity of debt on the asset's future cashflows then implies that, fixing a given security, $s(y) = \min\{y, D\}$, more accurate information improves the liquidity supplier's valuation for that security and hence the gains from trade. In other words, despite the risk neutrality of the liquidity supplier's utility function, the geometry of debt implies that increasing the accuracy of his private signal has an effect similar to a reduction in uncertainty for a risk-averse agent.

Formally, we prove below that, fixing an arbitrary debt menu (with face values monotonically ordered as implied by incentive compatibility), the gains from trade increase as the liquidity supplier's private signal becomes more accurate.

Proposition 3. *Consider an arbitrary menu of debt contracts characterized by the set of (monotone) face values $\{D(\omega)\}_{\omega\in\Omega}$. Suppose that $F'' \succeq_{Lehmann} F'$; then, $\mathbb{E}_{\mathbf{F}''_{\Phi}}\{\min\{\boldsymbol{y}, D(\boldsymbol{\omega})\}\} \geq \mathbb{E}_{\mathbf{F}'_{\Phi}}\{\min\{\boldsymbol{y}, D(\boldsymbol{\omega})\}\}$.*

The result is a direct consequence of property (i) in Proposition 2, above. Note that, for any nonnegative, nondecreasing function $D(\cdot)$, the function

$$\psi(y,\omega) \equiv \min\{y, D(\omega)\}$$

is supermodular in (y, ω). For any $\omega'' > \omega'$, the monotonicity of $D(\cdot)$ implies that

$$\psi(y, \omega'') - \psi(y, \omega') = \min\{y, D(\omega'')\} - \min\{y, D(\omega')\}$$

is nondecreasing in y. The fact that accuracy implies the supermodular order (part (i) in proposition (2)) implies that if $F'' \succeq_{\text{Lehmann}} F'$, then

$$\int_{\mathbb{R}_+ \times \Omega} \min\{y, D(\omega)\} \, \mathrm{d}\mathbf{F}''_\Phi(y, \omega) \geq \int_{\mathbb{R}_+ \times \Omega} \min\{y, D(\omega)\} \, \mathrm{d}\mathbf{F}'_\Phi(y, \omega).$$

Therefore, for any arbitrary debt menu with face values characterized by $D(\cdot)$, the liquidity supplier's ex ante valuation for the menu of securities increases as his information becomes more accurate.

Proposition 3 assumes a fixed menu of debt securities. However, as the accuracy of the liquidity supplier's private signal becomes more accurate, the issuer optimally responds by changing the face values of the debt securities. Our next result shows that the issuer's optimal menu monotonically decreases as we increase the accuracy of the liquidity supplier's signal.

Theorem 3. *Suppose Assumptions (1) and (2) hold. If $F'' \succeq_{\text{Lehmann}} F'$, then the respective optimal mechanisms under each experiment, characterized by the sets of face values $\{D'_*(\omega)\}_{\omega \in \Omega}$ and $\{D''_*(\omega)\}_{\omega \in \Omega}$, satisfy $D'_*(\omega) \geq D''_*(\omega)$, for all $\omega \in \Omega$.*

The formal proof is in the Appendix. We provide the intuition for the result here. Fix a marginal distribution Φ and an experiment F. Consider increasing the face value characterizing the contract designed for type ω, $D(\omega)$, by $\epsilon > 0$ small. The effect of such a variation on the issuer's revenue is approximately given by

$$\epsilon \frac{\partial}{\partial D(\omega)} \mathbb{E}(p(\boldsymbol{\omega}); F) = \epsilon \phi(\omega)(1 - F(D(\omega)|\omega)) - \epsilon(1 - \Phi(\omega)) \frac{\partial}{\partial \omega}(1 - F(D(\omega)|\omega)). \quad (7)$$

Indeed, with probability $\phi(\omega)$, the issuer faces liquidity supplier ω and obtains the additional revenue from increasing the contract $D(\omega)$ to $D(\omega) + \epsilon$ and selling at the fair valuation of type ω. The additional revenue is thus captured by $\epsilon \cdot (1 - F(D(\omega)|\omega))$. In turn, to prevent higher types $\tilde{\omega} > \omega$ from mimicking type ω, they need to be compensated with additional information rents. The type right next to type ω (type "$\omega + \mathrm{d}\omega$") observes a differential increment of his utility from mimicking in $\epsilon \cdot \frac{\partial}{\partial \omega}(1 - F(D(\omega)|\omega))$. The issuer needs to give up informational rent to all types above ω. Thus, the loss in revenue equates to $\epsilon \cdot (1 - \Phi(\omega)) \frac{\partial}{\partial \omega}(1 - G(D(\omega)|\omega))$.

The main theoretical insight of the proof is that because of the geometry of debt, the marginal incentive to increase the face value of a debt security, captured by $\frac{\partial}{\partial D(\omega)}\mathbb{E}\left(p\left(\boldsymbol{\omega}\right);F\right)$, has the single crossing property in the Lehmann order. That is, if for some experiment F', the issuer does not have an incentive to increase $D\left(\omega\right)$ (i.e., $\frac{\partial}{\partial D(\omega)}\mathbb{E}\left(p\left(\boldsymbol{\omega}\right);F'\right)\leq 0$), then for any $F''\succeq_{\text{Lehmann}}F'$, the issuer does not have incentives to raise $D\left(\omega\right)$ (i.e., $\frac{\partial}{\partial D(\omega)}\mathbb{E}\left(p\left(\boldsymbol{\omega}\right);F''\right)\leq 0$).

Put differently, when the experiment linking cashflows $\boldsymbol{y}$ and the liquidity supplier's information $\boldsymbol{\omega}$ becomes more accurate, information rents become too expensive. To prevent higher types from mimicking lower types, the optimal contracts designed for the latter must be distorted to a larger extent. This means that for each liquidity supplier type ω, the face value of the contract designed for him, $D(\omega)$, is weakly smaller when the accuracy of his signal is higher.

4.4 Information, Efficiency, and Funding Capacity

4.4.1 Gains from Trade

A natural conjecture about the consequences of Theorem 3 is that more information asymmetry, as captured by a more accurate private signal, is detrimental to efficiency. After all, as the liquidity supplier becomes more informed, the issuer optimally sells smaller securities to reduce information rents. The liquidity supplier is the efficient holder of the asset's future cashflows (as she is more patient than the issuer); therefore, efficiency might be compromised. However, as argued above in Proposition 3, there exists a novel effect associated with the geometry of the optimal securities. The overall effect is thus ambiguous and depends on the underlying distributions Φ and experiment F.

To formally see this, consider the liquidity supplier's ex ante valuation of the optimal securities when the accuracy of the liquidity supplier's signal is captured by experiment F. That is,

$$\begin{aligned}\mathbb{E}\left(\min\left\{\boldsymbol{y},D_{*}\left(\boldsymbol{\omega};F\right)\right\}\right) &= \int_{\Omega}\int_{0}^{\infty}\min\left\{y,D_{*}\left(\omega;F\right)\right\}\mathrm{d}F\left(y|\omega\right)\mathrm{d}\Phi\left(\omega\right)\\ &= \int_{\Omega}\left\{\int_{0}^{D_{*}(\omega;F)}y\mathrm{d}F\left(y|\omega\right)+D_{*}\left(\omega;F\right)\left(1-F\left(D_{*}\left(\omega;F\right)|\omega\right)\right)\right\}\mathrm{d}\Phi\left(\omega\right)\\ &= \int_{\Omega}\left\{\int_{0}^{D_{*}(\omega;F)}\left(1-F\left(y|\omega\right)\right)\mathrm{d}y\right\}\mathrm{d}\Phi\left(\omega\right). \qquad (8)\end{aligned}$$

Increasing the accuracy of experiment F has two effects. On the one hand, as implied by

Theorem 3, when F becomes more accurate, $D_*(\omega; F)$ decreases for all ω. This is the direct effect of selling smaller securities, which reduces the gains from trade. However, increasing the accuracy of F increases the liquidity supplier's valuation for concave securities. In general, the overall effect of increasing asymmetric information among the issuer and the liquidity supplier has an ambiguous effect on the gains from trade.

4.4.2 Revenue and Funding Capacity

Similarly, the effect of increasing the accuracy of the liquidity supplier's private signal is ambiguous for the issuer's expected revenue (i.e., $\mathbb{E}\{p_*(\boldsymbol{\omega}) + \delta(\boldsymbol{y} - s_*(\boldsymbol{y}|\boldsymbol{\omega}))\}$) and her funding capacity, which is captured by the expected proceeds using the optimal mechanism (i.e., $\mathbb{E}\{p_*(\boldsymbol{\omega})\}$). Because in the model both the issuer's revenue and her funding capacity perfectly commove with the accuracy of the private signal, we state our results focusing on the first item. Our results, however, readily extend to the issuer's funding capacity.

On top of the two effects already described above, there is generally a third effect that accounts for the change in information rents as the accuracy of the signal improves. Thus, whether a more accurate private signals increases the issuer's funding capacity ultimately depends on the balance of the three effects: In equilibrium, (a) she sells smaller securities (Theorem 3), (b) the geometry of debt increases the liquidity supplier's expected value of the security (Proposition 3), and (c), for a fixed set of securities, the information rents vary.

Lemma 5. *Let $\Pi_*(F) \equiv \mathbb{E}\{p_*(\boldsymbol{\omega}) + \delta(\boldsymbol{y} - s_*(\boldsymbol{y}|\boldsymbol{\omega}))\}$ be the issuer's expected revenue when the liquidity supplier's private information is parametrized by experiment F. Then,*[24]

$$\Pi_*(F) = \delta\mathbb{E}\{\boldsymbol{y}\} + \int_\Omega \int_0^{D_*(\omega;F)} (1 - F(y|\omega)) \left\{ 1 - \delta - \left(\frac{1 - \Phi(\omega)}{\phi(\omega)} \right) \frac{\frac{\partial}{\partial\omega}(1 - F(y|\omega))}{1 - F(y|\omega)} \right\} \mathrm{d}y \mathrm{d}\Phi(\omega). \tag{9}$$

Equation (9) summarizes the three effects described above. Effect (a) is captured by the integration limit of the inner integral. For any ω, $D_*(\omega; F)$ decreases with a more informative

[24]In turn, the expected proceeds of the security sale $\mathbb{E}\{p_*(\boldsymbol{\omega})\}$ can be obtained by evaluating expression 9 at $\delta = 0$. That is,

$$\mathbb{E}\{p_*(\boldsymbol{\omega})\} = \int_\Omega \int_0^{D_*(\omega;F)} (1 - F(y|\omega)) \left\{ 1 - \delta - \left(\frac{1 - \Phi(\omega)}{\phi(\omega)} \right) \frac{\frac{\partial}{\partial\omega}(1 - F(y|\omega))}{1 - F(y|\omega)} \right\} \mathrm{d}y \mathrm{d}\Phi(\omega).$$

experiment, together with the fact that[25]

$$\left(1-F\left(y|\omega\right)\right)\left\{1-\delta-\left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right)\frac{\frac{\partial}{\partial\omega}\left(1-F\left(y|\omega\right)\right)}{1-F\left(y|\omega\right)}\right\}\geq 0,\forall y\in\left[0,D_{*}\left(\omega;F\right)\right],$$

jointly imply that the amount of funds raised decreases with informativeness.

The countervailing effect (b) is a consequence of the concavity of debt, which implies that for any given debt menu with face values captured by $D(\cdot)$, the gains from trade, i.e.,

$$\mathbb{E}\left(\min\left\{\boldsymbol{y},D\left(\boldsymbol{\omega}\right)\right\}\right)=\int_{\Omega}\left\{\int_{0}^{D(\omega)}\left(1-F\left(y|\omega\right)\right)\mathrm{d}y\right\}\mathrm{d}\Phi\left(\omega\right),$$

are monotone in the informativeness of F (Proposition 3). Finally, effect (c) is obtained from the fact that

$$1-\delta-\left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right)\frac{\frac{\partial}{\partial\omega}\left(1-F\left(y|\omega\right)\right)}{1-F\left(y|\omega\right)}$$

is strictly decreasing in the informativeness of F (see the arguments establishing Theorem 3).

As a result, the overall effect of increasing the accuracy of the private signal on the issuer's revenue is also ambiguous. As suggested by Example 1, this ambiguous effect on both gains from trade and revenue does not manifest in the classical screening problem where the issuer is constrained to sell pure equity, a linear instrument of the asset's cashflows. The new countervailing effect materializes in our environment with the introduction of flexible security design. Example 1 shows that effect (b) can be large with respect to effects (a) and (c) and ultimately prevail. However, one can find conditions under which the result is reversed. Lemma 8 in the Appendix provides sufficient conditions under which effect (a) and (c) dominate effect (b), thereby implying that the issuer's revenue and funding capacity suffer when trading with better-informed liquidity suppliers.

Finally, we further note that the novel effect captured by (b) is quite different from previous

[25] To see this more clearly, let

$$\zeta\left(y,\omega\right)\equiv\left(1-\delta\right)\left(1-F\left(y|\omega\right)\right)-\left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right)\frac{\partial}{\partial\omega}\left(1-F\left(y|\omega\right)\right).$$

The definition of $D_{*}\left(\omega;F\right)$ implies that $\zeta\left(D_{*}\left(\omega;F\right),\omega\right)=0$, for all $\omega\in\Omega$. Moreover,

$$\begin{aligned}\frac{\partial}{\partial y}\zeta\left(y,\omega\right) &= -\left(1-\delta\right)f(y|\omega)+\left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right)\frac{\partial}{\partial\omega}f(y|\omega).\\ &< -f(y|\omega)\left(1-\delta-\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\frac{\frac{\partial}{\partial\omega}f(y|\omega)}{f(y|\omega)}\right)<0,\ \forall y<D_{*}\left(\omega;F\right).\end{aligned}$$

results. First, it is distinct from the celebrated *linkage principle* (Milgrom and Weber [1982]), which arises in common value auctions, and according to which the seller of the asset prefers to reduce the extent of asymmetric information to minimize the winner's curse. In our case, with a single liquidity supplier, the linkage principle does not apply. Further, Ottaviani and Prat [2001] discuss two reasons why a seller screening a buyer has incentives to reduce the degree of asymmetric information between them. First, contrary to our comparative statics exercise where we modify the accuracy of the liquidity supplier's private signal, they assume that any information publicly revealed to the buyer is also observed by the seller. This means that public announcements dilute the informational advantage of the buyer thereby increasing the seller's revenue. Second, in contrast to our assumption that prices are paid in period 1 and cannot be made contingent on the cashflow realizations, their seller can contract on the information publicly revealed. This last effect is reminiscent of the standard result in Crémer and McLean [1988] who show that a seller benefits from having access to a signal that correlates with the buyer's private information. Our result, in contrast, is a consequence of the geometry of financial securities. We argue that accounting for this effect is crucial for welfare analysis and should not be ignored when designing regulations.

5 Conclusions

We study how the flexibility of designing financial securities can help liquidity-constrained asset owners raise liquid funds from informed investors. We show, perhaps counterintuitively, that exploiting the information sensitivity of the securities within the menu is generally ineffective at screening the liquidity suppliers' private information and is dominated by simple debt menus. We show that information-insensitive securities allow the issuer to minimize the liquidity suppliers' information rents and therefore allow the former to raise larger amounts of funds.

Our contribution has an important methodological component. We show how, using the logic of a well-designed replication argument and a suitable generalization of the monotonicity condition, the standard tools in the screening literature can be extended to the rich environment of financial securities, i.e., infinite-dimensional objects. One of the advantages of our direct mechanism approach vis-a-vis former results in the literature is the characterization of the optimal mechanism with securities and the liquidity supplier's information rents within the mechanism. We leverage our characterization to perform novel comparative statics with respect to the primitives of the informational environment (e.g., the extent of agents' asymmetric information) and the issuer's ability to raise external funding. We show that as the liquidity supplier's private signal becomes more accurate, the issuer optimally designs a menu of smaller debt securities. Interestingly, the overall effect on surplus and the issuer's ability to raise funds remains ambiguous. Our results underscore the fact that, contrary to other type of goods, financial securities have a geometry associated with them which appears crucial for welfare analysis and should be accounted for when designing financial regulation.

The results in this paper are worth extending in several directions. The analysis assumes a single liquidity supplier. It would be interesting to generalize our direct mechanism approach to the case with multiple bidders and provide a general characterization of the liquidity suppliers' information rents in such a case.[26]

In our paper, we show how the optimal mechanism with securities change as we change the liquidity supplier's exogenous private information. It is reasonable to conjecture that in practice, asset owners can manipulate the information asymmetry with respect to the liquidity supplier. What is the optimal mechanism with securities when the issuer can design both the security and the information structure? Inostroza and Tsoy [2022] make progress in this direction and show that an issuer who can also design signal realizations prefers to sell highly information-sensitive securities.

Appendix A: Proofs Section 3 (Optimality of Debt Menus)

Proof of Proposition 1. To see (1), note that for $i, j \in \{1, ..., N\}$, with $i < j$,

$$\begin{aligned} U_{\mathcal{M}}(\omega_j; \omega_j) = \mathbb{E}_j(u_j(s_j)) - p_j &\geq U_{\mathcal{M}}(\omega_i; \omega_j) \\ &= \mathbb{E}_j(u_j(s_i)) - p_i \\ &> \mathbb{E}_i(u_i(s_i)) - p_i. \\ &\geq 0 \end{aligned}$$

where the first inequality follows from $[\mathrm{IC}_{j,i}]$, the second inequality is implied by FOSD (in turn implied by MLRP) and the monotonicity of the security, and the last inequality follows from $[\mathrm{IR}_i]$.

Next, to see property (2), let

$$\xi_{\mathcal{M}} \equiv \min_{n \in \{1,...,N\}} U_{\mathcal{M}}(\omega_n; \omega_n) = U_{\mathcal{M}}(\omega_1; \omega_1),$$

[26] DeMarzo et al. [2005] follow an indirect approach and find the optimal solution within a fairly large set of mechanisms. However, their solution, i.e., first-price auction in call-options, can be dominated with nonstandard mechanisms.

where the equality is a consequence of property (1). Suppose that $\xi_{\mathcal{M}} > 0$. Then, define $\tilde{\mathcal{M}} = \{\tilde{s}_n(\cdot), \tilde{p}_n\}_{n=1}^N$ as follows. Let $\tilde{s}_n \equiv s_n$, and $\tilde{p}_n \equiv p_n + \xi_{\mathcal{M}}$ for all $n \in \{1, ..., N\}$. Increasing the prices by the same amount to all types does not spoil incentive compatibility. Moreover, increasing the transfers by $\xi_{\mathcal{M}} > 0$ implies that individual rationality constraints are satisfied. The new mechanism raises more funds than the original mechanism while it preserves the securities offered to the liquidity supplier; therefore, $\tilde{\mathcal{M}}$ strictly dominates $\mathcal{M}$.

Finally, to see property (3), suppose that $\mathbb{P}\{\boldsymbol{y} - s_N(\boldsymbol{y}) > 0\} > 0$. Consider the mechanism $\hat{\mathcal{M}} = \{\hat{s}_n(\cdot), \hat{p}_n\}_{n=1}^N$ where $\hat{s}_j \equiv s_j$ and $\hat{p}_j \equiv p_j$ for all $j \in \{1, ..., N-1\}$, $\hat{s}_N(y) \equiv y$ for all y and $\hat{p}_N \equiv p_N + \varphi_N \mathbb{E}_N(y - s_N(y))$. By construction, under the new mechanism $\hat{\mathcal{M}}$, the utility of type ω_N remains unchanged, whereas the utility of any other type ω_j who chooses to report ω_N is strictly smaller than under $\mathcal{M}$. In fact,

$$
\begin{aligned}
U_{\hat{\mathcal{M}}}(\omega_N; \omega_j) &= \mathbb{E}_j(u_j(\hat{s}_N)) - \hat{p}_N \\
&= \mathbb{E}_j(u_j(s_N)) - p_N + \underbrace{(\varphi_j \mathbb{E}_j(y - s_N(y)) - \varphi_N \mathbb{E}_N(y - s_N(y)))}_{<0} \\
&< U_{\mathcal{M}}(\omega_N; \omega_j),
\end{aligned}
$$

where the inequality obtains from the fact that (i) $y - s_N(y)$ is monotone and (ii) $[\boldsymbol{y}|\boldsymbol{\omega} = \omega_N] \succ_{\text{MLRP}} [\boldsymbol{y}|\boldsymbol{\omega} = \omega_j]$. Therefore, the new mechanism $\hat{\mathcal{M}}$ is feasible. Finally, note that the issuer is strictly better off since

$$
\begin{aligned}
\mathbb{E}_N\{v(\boldsymbol{y}, \hat{p}_N, \hat{s}_N)\} = \hat{p}_N &= p_N + \varphi_N \mathbb{E}_N(\boldsymbol{y} - s_N(\boldsymbol{y})) \\
&> p_N + \delta \mathbb{E}(\boldsymbol{y} - s_N(\boldsymbol{y})). \\
&= \mathbb{E}_N\{v(\boldsymbol{y}, p_N, s_N)\},
\end{aligned}
$$

where the strict inequality follows from assumption (1); therefore, $\sum_i \mathbb{E}_i\{v(\boldsymbol{y}, \hat{p}_i, \hat{s}_i)\} \phi_i > \sum_i \mathbb{E}_i\{v(\boldsymbol{y}, p_i, s_i)\} \phi_i$. $\square$

Proof of Theorem 1

We divide the proof of the theorem into 3 steps. Step 1 shows that when, for a given type ω_i, the security s_i is debt, then for any $k > j > i$, the local downward incentive constraints [$\text{IC}_{k,j}$] and [$\text{IC}_{j,i}$] imply the global downward constraint [$\text{IC}_{k,i}$]. Step 2 builds on this result to show that, in this case, the incentive constraint [$\text{IC}_{i+1,i}$] must necessarily bind at the optimal mechanism. Step 3 finally proves that, starting from a mechanism where some of the securities are not debt, one can construct an alternative mechanism where the nondebt securities are changed by their payoff equivalent debt security and that this does not compromise upward incentive constraints while relaxing the binding downward incentive constraints.

Step 1. We show that at any oriented mechanism, if for some $i \in \{1, ..., N-2\}$ the security s_i is debt, then for any $k > j > i$, the local downward incentive constraints [IC$_{k,j}$] and [IC$_{j,i}$] imply the downward global constraint [IC$_{k,i}$]. This means that global downward constraints are redundant and can be ignored.

Proposition 4. [LOCAL CONSTRAINTS IMPLY GLOBAL CONSTRAINTS]*Let $\mathcal{M} = \{s_i, p_i\}_{i=1}^{N}$ be an oriented mechanism. Assume that, for some $i < N$, $s_i = \min\{y, D_i\}$ for ($\lambda-$almost all) y, with $D_i > 0$. Suppose that, for any $i, j, k \in \{1, ..., N\}$ with $k > j > i$, (i) [$IC_{j,i}$], (ii) [$IC_{k,j}$], and (iii) [IR_i] jointly hold; then, [$IC_{k,i}$] must also hold.*

Proof. That incentive constraint [IC$_{j,i}$] holds implies that

$$\mathbb{E}_j \{u_j (s_j)\} - p_j \geq \mathbb{E}_j \{u_j (s_i)\} - p_i. \tag{10}$$

Next, we show that

$$\mathbb{E}_k \{s_j - s_i\} \geq \mathbb{E}_j \{s_j - s_i\}. \tag{11}$$

To see this, note first that $\mathcal{M}$ is oriented only if $\mathbb{E}_j \{s_i - s_j\} \leq 0$. Next, let $\gamma \geq 1$ be implicitly defined by

$$\mathbb{E}_j \{\gamma s_i - s_j\} = 0. \tag{12}$$

That s_i is a debt security, together with the fact that $\gamma \geq 1$, imply that $\gamma s_i - s_j$ satisfies SCFA. This last observation, together with the fact that $[\boldsymbol{y}|\boldsymbol{\omega} = \omega_k] \succ_{\text{MLRP}} [\boldsymbol{y}|\boldsymbol{\omega} = \omega_j]$, jointly imply that lemma (1) applies and therefore

$$\mathbb{E}_k \{\gamma s_i - s_j\} \leq 0. \tag{13}$$

As a result, we conclude that

$$\begin{aligned} \mathbb{E}_j \{s_i - s_j\} - \mathbb{E}_k \{s_i - s_j\} &= \underbrace{\mathbb{E}_j \{\gamma s_i - s_j\}}_{=0 \text{ from (12)}} - \underbrace{\mathbb{E}_k \{\gamma s_i - s_j\}}_{\leq 0 \text{ from (13)}} \\ &+ (\gamma - 1) \times \underbrace{(\mathbb{E}_k \{s_i\} - \mathbb{E}_j \{s_i\})}_{\geq 0 \text{ from MLRP}} \\ &\geq 0, \end{aligned}$$

which proves the inequality in (11).

Combining inequalities (10) and (11), and using the fact that $\mathbb{E}_j \{s_j - s_i\} \geq 0$ (by orientation), we then obtain that type ω_k weakly prefers the security designed for type ω_j over the one designed for type ω_i. That is,

$$\mathbb{E}_k \{u_k (s_j)\} - p_j \geq \mathbb{E}_k \{u_k (s_i)\} - p_i. \tag{14}$$

Finally, the fact that the local incentive constraint [IC$_{k,j}$] is also satisfied implies that

$$\mathbb{E}_k \{u_k (s_k)\} - p_k \geq \mathbb{E}_k \{u_k (s_j)\} - p_j. \tag{15}$$

Combining (14) and (15), we thus conclude that type ω_k weakly prefers his contract over the one designed for type ω_i. That is,

$$\mathbb{E}_k \{u_k (s_k)\} - p_k \quad \geq \quad \mathbb{E}_k \{u_k (s_i)\} - p_i,$$

and hence [IC$_{k,i}$] is satisfied. □

This completes the proof of Step 1. ■

Step 2. We leverage on the result in Step 1 to show that, when s_i is a debt contract, then it is without loss to restrict attention to mechanisms for which the local downward incentive constraints [IC$_{i+1,i}$] bind.

Corollary 2. *Consider any oriented mechanism $\mathcal{M} = \{s_n, p_n\}_{n=1}^N$ for which there exists some $j \in \{1, ..., N-2\}$ such that (i) $s_j = \min\{y, D_j\}$ for ($\lambda-$almost all) y, with $D_j > 0$, and (ii) [$IC_{j+1,j}$] does not bind. Then, $\mathcal{M}$ is strictly dominated by another feasible and oriented mechanism for which [$IC_{j+1,j}$] does bind.*

Proof. Suppose by contradiction that there exists some $j \in \{1, ..., N-2\}$ such that $s_j = \min\{y, D_j\}$ for ($\lambda-$almost all) y, with $D_j > 0$, for which [IC$_{j+1,j}$] is satisfied with slackness. The steps establishing proposition 4 then imply that, for any k, i with $k \geq j+1 > i$, [IC$_{k,i}$] is also slack. That is,

$$\forall k, i \text{ with } k \geq j+1 > i, U_{\mathcal{M}} (\omega_k; \omega_k) - U_{\mathcal{M}} (\omega_i; \omega_k) > 0. \tag{16}$$

The issuer can then construct an alternative mechanism $\mathcal{M}^+ = \left\{s_i^+, p_i^+\right\}_{i=1}^N$ that strictly dominates $\mathcal{M}$. In fact, for any $l \in \{1, ..., N\}$, let $s_l^+ \equiv s_l$. For any $h \leq j$, let $p_h^+ \equiv p_h$, and for any $h > j$, let $p_h^+ \equiv p_h + \epsilon$, where $\epsilon > 0$ is small and chosen so that incentive constraints are not spoiled. Note that, for any $k'', k' \in \{j+1, ..., N\}$, with $k'' \geq k'$,

$$\mathbb{E}_{k''}\left\{u_{k''}\left(s_{k''}^{+}\right)\right\}-p_{k''}^{+}-\left(\mathbb{E}_{k'}\left\{u_{k'}\left(s_{k'}^{+}\right)\right\}-p_{k'}^{+}\right)=\mathbb{E}_{k''}\left\{u_{k''}\left(s_{k''}\right)\right\}-p_{k''}-\left(\mathbb{E}_{k'}\left\{u_{k'}\left(s_{k'}\right)\right\}-p_{k'}\right)\geq 0,$$

and therefore [IC$_{k'',k'}$] trivially holds. That, for any $i'', i' \in \{1, ..., j\}$, [IC$_{i'',i'}$] holds, follows from the facts that $\mathcal{M}$ is feasible and that, for any $i < j$, $\left(s_i^+, p_i^+\right) = (s_i, p_i)$. Finally, that under $\mathcal{M}^+$, for any k, i with $k \geq j+1 > i$, [IC$_{k,i}$] holds for any

$$\epsilon \in \left(0, \min_{k,i:k\geq j+1>i}\left(U_{\mathcal{M}}\left(\omega_k;\omega_k\right)-U_{\mathcal{M}}\left(\omega_i;\omega_k\right)\right)\right)$$

follows from the observation in (16). □

This completes the proof of Step 2. ■

Step 3. We finally show that for any oriented mechanism $\mathcal{M}$, if we let $i+1 \leq N$ be the smallest type for whom his security s_{i+1} is not debt, we can improve upon the issuer's payoff by swapping security s_{i+1} for the payoff-equivalent (according to ω_{i+1}'s beliefs) debt security s_{i+1}^D without spoiling upward incentive constraints [IC$_{h,i+1}$], for $h \leq i$.

First, we prove an intermediate result.

Proposition 5. [Pooling]*Let $\mathcal{M} = \{s_i, p_i\}_{i=1}^N$ be a feasible mechanism. Assume that, for some $i < N$, $s_i = \min\{y, D_i\}$ with $D_i > 0$, for ($\lambda-$almost all) y. Suppose that, for some $j > i$, [$IC_{i,j}$] and [$IC_{j,i}$] are both binding. If $s_i(y) \neq s_j(y)$ over a set with positive λ-measure, then there exists another mechanism $\hat{\mathcal{M}} = \{\hat{s}_i, \hat{p}_i\}_{i=1}^N$ with $\hat{s}_i = \hat{s}_j$ that strictly dominates $\mathcal{M}$.*

Proof. That [IC$_{i,j}$] and [IC$_{j,i}$] are both binding implies that

$$\varphi_i \mathbb{E}_i\{s_i - s_j\} = p_i - p_j = \varphi_j \mathbb{E}_j\{s_i - s_j\}. \tag{17}$$

Assume next that $s_i \neq s_j$ over a set with positive λ-measure. We show that $\mathbb{E}_i\{s_i - s_j\} > 0$. Suppose by contradiction that $\mathbb{E}_i\{s_i - s_j\} \leq 0$. This implies that there must exist $\gamma \geq 1$ so that

$$\mathbb{E}_i\{\gamma s_i - s_j\} = 0. \tag{18}$$

The fact that s_i is a debt security, together with the assumption that $s_i \neq s_j$, then jointly imply that $\gamma s_i - s_j$ satisfies SSCFA, and therefore, lemma 1 implies that

$$\mathbb{E}_j\{\gamma s_i - s_j\} < 0. \tag{19}$$

Thus,

$$\begin{aligned}\mathbb{E}_i\{s_i - s_j\} - \mathbb{E}_j\{s_i - s_j\} &= \underbrace{\mathbb{E}_i\{\gamma s_i - s_j\}}_{=0 \text{ from } (18)} - \underbrace{\mathbb{E}_j(\gamma s_i - s_j)}_{<0 \text{ from } (19)} \\ &+ (\gamma - 1) \times \underbrace{(\mathbb{E}_j\{s_i\} - \mathbb{E}_i\{s_i\})}_{>0 \text{ from MLRP}} \\ &> 0,\end{aligned}$$

which contradicts equation (17). Thus, $\mathbb{E}_i\{s_i - s_j\} > 0$.

Assume then that $s_i \neq s_j$ over a set with positive λ-measure and that $\mathbb{E}_i\{s_i - s_j\} > 0$. Equation (17) then implies that (a) $\mathbb{E}_j\{s_i - s_j\} > 0$ and (b) $p_i > p_j$. This, in turn, means that there exists an alternative mechanism, $\hat{\mathcal{M}}$, identical to $\mathcal{M}$ except for the fact that it offers contract $(\hat{s}_j, \hat{p}_j) = (s_i, p_i)$ to type ω_j. In other words, $\hat{\mathcal{M}}$ deletes the contract offered to type ω_j under the mechanism $\mathcal{M}$ and replaces it by the debt contract offered to type ω_i. Clearly, the new mechanism $\hat{\mathcal{M}}$ is still feasible as the contract (s_i, p_i) was already available under $\mathcal{M}$. Moreover, $\hat{\mathcal{M}}$ strictly dominates $\mathcal{M}$ as $\hat{p}_j = p_i > p_j$ and $\hat{p}_l = p_l$ for any $l \neq j$. This proves the proposition. □

Next, consider a candidate oriented mechanism $\mathcal{M} = \{s_n, p_n\}_{n=1}^{N}$. Proposition 1 and Lemmas 2 - 4 jointly imply that it is without loss of optimality to restrict attention to mechanisms satisfying (A) $s_1 = \min\{y, D_1\}$ for ($\lambda-$almost all) y; (B) $s_N = y$ for ($\lambda-$almost all) y ; (C) $p_1 = \mathbb{E}_1\{s_1\}$; (D) for any j, k with $j < k$, $\mathbb{E}_k\{s_k - s_j\} \geq 0$ (lemma 3). Assume further that there exists $i \geq 1$, so that (E) for any $h \leq i$, $s_h = \min\{y, D_h\}$ for ($\lambda-$almost all) y, and, by means of corollary (2) that (F) for any $h \leq i$, $[\text{IC}_{h+1,h}]$ binds. Finally suppose that s_{i+1} is not a debt contract. We show that we can construct a new oriented mechanism $\tilde{\mathcal{M}} \equiv \{\tilde{s}_h, \tilde{p}_h\}$ that strictly dominates $\tilde{\mathcal{M}}$. In fact, for any $h \neq i+1$, let $(\tilde{s}_h, \tilde{p}_h) \equiv (s_h, p_h)$, and let $\tilde{s}_{i+1} \equiv \min\{y, D_{i+1}\}$ where D_{i+1} is such that $\mathbb{E}_{i+1}\{\tilde{s}_{i+1} - s_{i+1}\} = 0$ and $\tilde{p}_{i+1} \equiv p_{i+1}$. Note that the fact that, under $\mathcal{M}$, $[\text{IC}_{i+1,i}]$ binds implies that

$$\tilde{p}_{i+1} = p_{i+1} = \mathbb{E}_{i+1}\{u_{i+1}(s_{i+1}) - u_{i+1}(s_i)\} + p_i = \mathbb{E}_{i+1}\{u_{i+1}(\tilde{s}_{i+1}) - u_{i+1}(\tilde{s}_i)\} + \tilde{p}_i. \quad (20)$$

Claim 1. *Under the new mechanism $\tilde{\mathcal{M}}$, $[IR_{i+1}]$ and $[IC_{i+1,h}]$ hold for any $h \neq i+1$.*

Proof. The proof follows from noting that, by construction, for any $h \neq i+1$, $(\tilde{s}_h, \tilde{p}_h) = (s_h, p_h)$ and the fact that the new contract makes type ω_{i+1} indifferent between the new and the former contract. That is, $U_{\tilde{\mathcal{M}}}(\omega_{i+1}; \omega_{i+1}) = U_{\mathcal{M}}(\omega_{i+1}; \omega_{i+1})$. The properties then are inherited from mechanism $\mathcal{M}$. *q.e.d.*

Claim 2. *Under the new mechanism $\tilde{\mathcal{M}}$, for any $h \leq i$, $[IC_{h,i+1}]$ holds.*

Proof. Property (D) above, together with the construction of $\tilde{\mathcal{M}}$, jointly imply that, for any $h \leq i$,

$$\mathbb{E}_{i+1}\left\{\tilde{s}_{i+1} - \tilde{s}_h\right\} = \mathbb{E}_{i+1}\left\{s_{i+1} - s_h\right\} \geq 0,$$

and therefore that $D_{i+1} \geq D_h$. This observation, in turn, implies that $\tilde{s}_{i+1} - \tilde{s}_h$ is nondecreasing and, as a result, for any $h \leq i$,

$$\begin{aligned}
\mathbb{E}_h\left\{u_h\left(\tilde{s}_{i+1}\right) - u_h\left(\tilde{s}_h\right)\right\} &= \mathbb{E}_h\left\{u_h\left(\tilde{s}_{i+1}\right) - u_h\left(\tilde{s}_i\right)\right\} + \mathbb{E}_h\left\{u_h\left(\tilde{s}_i\right) - u_h\left(\tilde{s}_h\right)\right\} \\
&\leq \mathbb{E}_{i+1}\left\{u_{i+1}\left(\tilde{s}_{i+1}\right) - u_{i+1}\left(\tilde{s}_i\right)\right\} + \mathbb{E}_h\left\{u_h\left(\tilde{s}_i\right) - u_h\left(\tilde{s}_h\right)\right\} \\
&= \tilde{p}_{i+1} - \tilde{p}_i + \mathbb{E}_h\left\{u_h\left(\tilde{s}_i\right) - u_h\left(\tilde{s}_h\right)\right\} \\
&\leq \tilde{p}_{i+1} - \tilde{p}_i + \tilde{p}_i - \tilde{p}_h \\
&= \tilde{p}_{i+1} - \tilde{p}_h
\end{aligned}$$

where the first inequality follows from MLRP and the monotonicity φ_n, the second equality follows from using the result in equation (20), the second inequality obtains from the fact that, for any $h \leq i$, $[\mathrm{IC}_{h,i}]$ holds, which is inherited from the feasibility of $\mathcal{M}$. As a result, under the mechanism $\tilde{\mathcal{M}}$, for any $h \leq i$, $[\mathrm{IC}_{h,i+1}]$ holds. *q.e.d.*

Claim 3. *Under the new mechanism $\tilde{\mathcal{M}}$, for any $k > i+1$, $[IC_{k,i+1}]$ holds with slackness.*

The construction of $\tilde{\mathcal{M}}$, together with the fact that $s_{i+1} - \tilde{s}_{i+1}$ satisfies SSCFB (recall that $\tilde{s}_{i+1}$ is a debt security), jointly imply that by virtue of lemma 1, for any $k > i+1$,

$$\mathbb{E}_k\left\{s_{i+1} - \tilde{s}_{i+1}\right\} > 0. \tag{21}$$

The fact that, under $\mathcal{M}$, $[IC_{k,i+1}]$ *holds* thus implies that

$$\begin{aligned}
\mathbb{E}_k\left\{u_k\left(\tilde{s}_k\right)\right\} - \tilde{p}_k &= \mathbb{E}_k\left\{u_k\left(s_k\right)\right\} - p_k, \\
&\geq \mathbb{E}_k\left\{u_k\left(s_{i+1}\right)\right\} - p_{i+1} \\
&> \mathbb{E}_k\left\{u_k\left(\tilde{s}_{i+1}\right)\right\} - \tilde{p}_{i+1},
\end{aligned}$$

where the equality is by construction; the first inequality follows from the incentive compatibility of the original mechanism $\mathcal{M}$; the strict inequality follows from (21) above. This proves the claim. *q.e.d.*

Claims (1)-(3) then jointly imply that $\tilde{\mathcal{M}}$ is feasible. Furthermore, Claim (3) together with the steps establishing Lemma 4 jointly imply that, for any k, h with $k > i+1 \geq h$, $[\mathrm{IC}_{k,h}]$

does not bind. That is,

$$\forall k,h \text{ with } k>i+1\geq h, U_{\tilde{\mathcal{M}}}\left(\omega_k;\omega_k\right)-U_{\tilde{\mathcal{M}}}\left(\omega_h;\omega_k\right)>0. \tag{22}$$

This observation implies that, for any $k>i+1$, we can increase the transfers $\tilde{p}_k$ and still respect feasibility.

Rigorously, we can construct yet another feasible mechanism $\mathcal{M}^+=\{s_n^+,p_n^+\}_{i=1}^N$ that *strictly* dominates $\tilde{\mathcal{M}}$. In fact, for any $n\in\{1,...,N\}$, let $s_n^+\equiv s_n$, for any $h\leq i+1$, let $p_h^+\equiv\tilde{p}_h$, and for any $h>i+1$, let $p_h^+\equiv\tilde{p}_h+\epsilon$, where $\epsilon>0$ is small and chosen so that

$$\epsilon\in\left(0,\min_{k,h:k>i+1\geq h}\left(U_{\mathcal{M}}\left(\omega_k;\omega_k\right)-U_{\mathcal{M}}\left(\omega_i;\omega_k\right)\right)\right)$$

Note that, for any $k'',k'\in\{i+2,...,N\}$,

$$\mathbb{E}_{k''}\left(u_{k''}\left(s_{k''}^+\right)\right)-p_{k''}^+=\mathbb{E}_{k''}\left(u_{k''}\left(s_{k''}\right)\right)-p_{k''}\geq\mathbb{E}_{k'}\left(u_{k'}\left(s_{k'}\right)\right)-p_{k'}=\mathbb{E}_{k'}\left(u_{k'}\left(s_{k'}^+\right)\right)-p_{k'}^+,$$

and therefore $[\mathrm{IC}_{k'',k'}]$ trivially holds. That, for any $h'',h'\in\{1,...,i+1\}$, $[\mathrm{IC}_{h'',h'}]$ holds, follows from the fact that $\tilde{\mathcal{M}}$ is feasible and the fact that, for any $h\leq i+1$, $\left(s_h^+,p_h^+\right)=\left(s_h,p_h\right)$. Finally, that under $\mathcal{M}^+$, for any k,h with $k>i+1>h$, $[\mathrm{IC}_{k,h}]$ holds, follows from the choice of ϵ and the observation in (16). This completes Step 3 and formally proves the Theorem. □

Proof of Lemma 4.

The result in theorem 1 implies that we can restrict attention to mechanisms with only debt securities, that is, mechanisms satisfying that, for any n, $s_n=\min\{y,D_n\}$ for all y, where $D_n>0$ and is increasing in n, and $D_N=+\infty$. Moreover, Step 2 in the proof theorem 1 implies that we can restrict attention, without loss, to mechanisms satisfying that, for any $j>1$, $[\mathrm{IC}_{j,j-1}]$ binds. Finally, because of proposition 1, we assume that $p_1=\mathbb{E}\left\{\min\left\{\boldsymbol{y},D_1\right\}\right\}$. Together, these properties imply that

$$\mathbb{E}_j\left\{u_j\left(\min\left\{\boldsymbol{y},D_j\right\}\right)\right\}-p_j=\mathbb{E}_j\left\{u_j\left(\min\left\{\boldsymbol{y},D_{j-1}\right\}\right)\right\}-p_{j-1},$$

and therefore, for any $j > 1$,

$$\begin{aligned} p_j &= \mathbb{E}_j \left\{ u_j \left(\min \left\{ \boldsymbol{y}, D_j \right\} \right) \right\} - \mathbb{E}_j \left\{ u_j \left(\min \left\{ \boldsymbol{y}, D_{j-1} \right\} \right) \right\} + p_{j-1} \\ &= \left(\sum_{n>1}^{j} \mathbb{E}_n \left\{ u_n \left(\min \left\{ \boldsymbol{y}, D_n \right\} \right) \right\} - \mathbb{E}_n \left\{ u_n \left(\min \left\{ \boldsymbol{y}, D_{n-1} \right\} \right) \right\} \right) + \mathbb{E}_1 \left\{ u_1 \left(\min \left\{ \boldsymbol{y}, D_1 \right\} \right) \right\}. \end{aligned}$$

This further implies that the amount of funds raised is given by[27]

$$\begin{aligned} \sum_{j=1}^{N} p_n \phi_n &= \mathbb{E}_1 \left\{ u_1 \left(\min \left\{ y, D_1 \right\} \right) \right\} + \sum_{j>1} \phi_j \sum_{n>1}^{j} \mathbb{E}_n \left\{ u_n \left(\min \left\{ \boldsymbol{y}, D_n \right\} \right) \right\} - \mathbb{E}_n \left\{ u_n \left(\min \left\{ \boldsymbol{y}, D_{n-1} \right\} \right) \right\} \\ &= \mathbb{E}_1 \left\{ u_1 \left(\min \left\{ \boldsymbol{y}, D_1 \right\} \right) \right\} + \sum_{n>1}^{N} \left(\sum_{j=n}^{N} \phi_j \right) \left(\mathbb{E}_n \left\{ u_n \left(\min \left\{ \boldsymbol{y}, D_n \right\} \right) \right\} - \mathbb{E}_n \left\{ u_n \left(\min \left\{ \boldsymbol{y}, D_{n-1} \right\} \right) \right\} \right). \\ &= \sum_{n=1}^{N} \phi_n \int_0^{\infty} u_n \left(\min \left\{ y, D_n \right\} \right) \left(1 - \left(\frac{1 - \Phi_n}{\phi_n} \right) \left(\frac{f \left(y | \omega_{n+1} \right) - f \left(y | \omega_n \right)}{f \left(y | \omega_n \right)} \right) \right) \mathrm{d}F \left(y | \omega_n \right). \end{aligned}$$

As a result, the issuer's problem reduces to find an increasing sequence $(D_n)_{n=1}^N$ to maximize

$$\sum_{n=1}^{N} p_n \phi_n = \sum_{n=1}^{N} \phi_n \int_0^{\infty} u_n \left(\min \left\{ y, D_n \right\} \right) \left(1 - \left(\frac{1 - \Phi_n}{\phi_n} \right) \left(\frac{f \left(y | \omega_{n+1} \right) - f \left(y | \omega_n \right)}{f \left(y | \omega_n \right)} \right) \right) \mathrm{d}F \left(y | \omega_n \right),$$

as claimed.□

Proof of Lemma 2.

Assumptions 2 and 3 jointly imply that, for each n, the pointwise-optimal solution of the problem (that is, disregarding the monotonicity condition in (3)), given by

$$D_n^* \equiv \arg\max_{D} \quad \int_{\mathbb{R}_+} u_n \left(\min \left\{ y, D \right\} \right) \left(1 - R \left(y, \omega_n \right) \right) \mathrm{d}F \left(y | \omega_n \right), \tag{23}$$

is monotone in n. This follows from noting that under assumptions 2 and 3, the objective is supermodular (y, ω_n). Indeed, note that the fact that $u_n(x) = \varphi_n x + \nu_n$, implies that

$$\begin{aligned} \frac{\partial}{\partial D} \int_0^{\infty} u_n \left(\min \left\{ y, D \right\} \right) \left(1 - R \left(y, \omega_n \right) \right) \mathrm{d}F \left(y | \omega_n \right) &= \varphi_n \int_D^{\infty} \left(1 - R \left(y, \omega_n \right) \right) \mathrm{d}F \left(y | \omega_n \right), \\ &= \varphi_n \left(1 - F \left(y | \omega_n \right) - \int_D^{\infty} R \left(y, \omega_n \right) \mathrm{d}F \left(y | \omega_n \right) \right) \end{aligned}$$

[27]In the formula, with abuse notation and let $D_N = +\infty$ (claim 3 in Proposition 1).

When assumptions (2) and (3) hold, the last expression becomes monotone in ω_n. Hence, the pointwise-optimal solution D_n^* is necessarily nondecreasing (Milgrom and Shannon [1994]). This rules out the possibility of bunching.$\square$

Appendix B: Proof of Theorem 2

Discrete Case

Consider the relaxed program (6) where only the local incentive constraints are imposed. Denote as $\hat{\mathcal{M}} = \{\hat{s}_n, \hat{p}_n\}_{n=1}^N$ the solution of such a problem. The issuer's payoff under $\hat{\mathcal{M}}$ is weakly higher than under mechanism $\mathcal{M}^\star$ that solves program (1), because the set of constraints in the former program is a strict subset of the latter.

We argue that in the relaxed problem, local-downward incentive constraints must bind. Suppose by contradiction that a constraint $[\mathrm{IC}_{i+i,i}]$, $i \leq N-1$, does not bind. The issuer can construct an alternative mechanism $\mathcal{M}^\# = \left\{s_n^\#, p_n^\#\right\}_{n=1}^N$, identical to $\hat{\mathcal{M}}$ but where $p_n^\# = \hat{p}_n + \epsilon$ for all $n \geq i+1$, where

$$\epsilon = \mathbb{E}_{i+1}\left\{u_{i+1}\left(\hat{s}_{i+1}\right) - u_{i+1}\left(\hat{s}_i\right)\right\} - \left(\hat{p}_{i+1} - \hat{p}_i\right).$$

That is, ϵ is chosen so that the downward, local incentive compatibility constraint of type ω_{i+1} is binding. The new mechanism further increases the price to all types above $n \geq i+1$. The new mechanism is feasible. Indeed, for any $n \geq i+1$,

$$\begin{aligned} U_{\mathcal{M}^\#}\left(\omega_n;\omega_n\right) &= U_{\hat{\mathcal{M}}}\left(\omega_n;\omega_n\right) - \epsilon \\ &\geq U_{\hat{\mathcal{M}}}\left(\omega_{i+1};\omega_{i+1}\right) - \epsilon \\ &= U_{\hat{\mathcal{M}}}\left(\omega_i;\omega_{i+1}\right) \\ &> U_{\hat{\mathcal{M}}}\left(\omega_i;\omega_i\right) \geq 0, \end{aligned}$$

thereby implying that $\mathcal{M}^\#$ is individual rationality. Further, (local) incentive compatibility is in turn implied by construction. The new mechanism sells the same securities than $\hat{\mathcal{M}}$ at weakly larger prices and is therefore preferred by the issuer. This contradicts the optimality of $\hat{\mathcal{M}}$.

Next, we build on the fact that the constraints $[\mathrm{IC}_{i+i,i}]$ bind, $i \leq N-1$, to rewrite the issuer's payoff. Lemma 4 implies that

$$\sum_{n=1}^{N}\phi_n p_n=\sum_{n=1}^{N}\phi_n\int_0^\infty s_n\left(y\right)\left(1-\left(\frac{1-\Phi_n}{\phi_n}\right)\left(\frac{f_{n+1}\left(y\right)-f_n\left(y\right)}{f_n\left(y\right)}\right)\right)\mathrm{d}F_n\left(y\right).$$

Therefore, the maximizing the issuer's payoff, $\sum_{n=1}^{N}\phi_n\left(p_n+\delta\int_0^\infty\left(y-s_n(y)\right)\mathrm{d}F_n(y)\right)$, is equivalent to maximizing

$$V\left(\{s_n\}_{n=1}^N\right)\equiv\sum_{n=1}^{N}\phi_n\int_0^\infty s_n\left(y\right)\left(1-\delta-\left(\frac{1-\Phi_n}{\phi_n}\right)\left(\frac{f_{n+1}\left(y\right)-f_n\left(y\right)}{f_n\left(y\right)}\right)\right)\mathrm{d}F_n\left(y\right).$$

Lemma 6. *Consider an arbitrary incentive compatible mechanism $\mathcal{M}=\{s_n,p_n\}_{n=1}^N$. Suppose that there exists a nonempty set $\Omega^{\text{Non-debt}}\subseteq\{1,...,N\}$, such that, for any $n\in\Omega^{\text{Non-debt}}$, s_n is not a debt security. There exists a set of debt securities $\left\{s_n^{\#\#}\right\}_{n\in\Omega^{\text{Non-debt}}}$ so that the mechanism $\mathcal{M}^{\#\#}$ constructed from $\mathcal{M}$ by replacing s_n for $s_n^{\#\#}$ for all $n\in\Omega^{\text{Non-debt}}$, dominates $\mathcal{M}$ for the issuer.*

Proof. For any $n\in\Omega^{\text{Non-debt}}$, let

$$k_n(y;\delta)\equiv1-\delta-\left(\frac{1-\Phi_n}{\phi_n}\right)\left(\frac{f_{n+1}\left(y\right)-f_n\left(y\right)}{f_n\left(y\right)}\right).$$

The issuer's objective can then be written as

$$V\left(\{s_n\}_{n=1}^N\right)=\sum_{n=1}^{N}\phi_n\int_0^\infty s_n\left(y\right)k_n\left(y;\delta\right)\mathrm{d}F_n\left(y\right).$$

The fact that $f\left(y|\omega\right)$ satisfies MLRP implies that $k_n\left(\cdot;\delta\right)$ is monotonically decreasing and satisfies the SSCFA property. For each $n\in\{1,...,N\}$, let $y_n(\delta)$ be the unique solution to $k_n(y_n(\delta);\delta)=0$. From the definition of $D_n^\star$ in 5, we have that $y_n(\delta)>D_n^\star$, for all n. The constraint that all securities in $\mathcal{S}$ are nondecreasing, together with the fact that $k_n(y;\delta)<0$ for all $y>y_n\left(\delta\right)$, jointly imply that, any security $\tilde{s}(\cdot|\omega)$ in $\mathcal{S}$ which fails to be constant to the right of $y_n\left(\delta\right)$ is dominated by the security

$$s_n^{\#}(y)\equiv\tilde{s}_n(y)1\{y\leq y_n(\delta)\}+\tilde{s}_n(y_n(\delta))1\{y>y_n(\delta)\}.$$

Finally, the fact that $k_n(y;\delta)>0$ for all $y<y_n\left(\delta\right)$ implies that any security $\bar{s}_n(y)\in\mathcal{S}$,

also satisfying $\bar{s}_n(y) = d$ for all $y \geq y_n(\delta)$, is weakly dominated by the security

$$\begin{aligned} s^{\#\#}(y|\omega) &= \min\{y, d\} \cdot 1\{y < y_n(\delta)\} + d \cdot 1\{y \geq y_n(\delta)\}. \\ &= \min\{y, d\}. \end{aligned}$$

This proves the lemma.□

Finally, consider the function

$$\chi(n, D) \equiv \int_0^D y k_n(y;\delta) \mathrm{d}F_n(y) + D \int_D^\infty k_n(y;\delta) \mathrm{d}F_n(y).$$

We show that $\chi(n, D)$ is supermodular. Indeed,

$$\begin{aligned} \frac{\partial \chi}{\partial D}(n, D) &= \int_D^\infty k_n(y;\delta) \mathrm{d}F_n(y), \\ &= (1-\delta)(1 - F_n(D)) - \left(\frac{1-\Phi_n}{\phi_n}\right)\Delta_n. \end{aligned}$$

and therefore, for any $n \in \{1, ..., N-1\}$,

$$\frac{\partial \chi}{\partial D}(n+1, D) - \frac{\partial \chi}{\partial D}(n, D) = \underbrace{(1-\delta)\Delta_n}_{>0 \text{ (FOSD)}} - \underbrace{\left(\left(\frac{1-\Phi_{n+1}}{\phi_{n+1}}\right)\Delta_{n+1} - \left(\frac{1-\Phi_n}{\phi_n}\right)\Delta_n\right)}_{<0 \text{ (Assumptions 2,3)}} > 0.$$

Topkis Theorem then implies that the value of D_n that pointwise maximizes $\chi(n, D)$, $D_n^\star$, must be increasing in n. This further implies that the set of debt securities $\{\hat{s}_n = \min\{y, D_n^\star\}\}_{n=1}^N$ is oriented. The orientation of $\{\hat{s}_n\}_{n=1}^N$ in turn implies that,

Finally, by restricting attention to oriented mechanisms, we can use the same arguments establishing Step 1 in the proof of Theorem 1 to show that any mechanism respecting local, downward incentive constraints is globally incentive compatible (i.e., the incentive constraints $[\mathrm{IC}_{i,j}]$, $i \in \{1, ..., N\}$, $|j - i| > 1$ are also satisfied). Thus, the solution to the relaxed program $\hat{\mathcal{M}}$ is also feasible at the original program (1) and must therefore be optimal.

Continuum Case

Re-statement of the Theorem.

Theorem 4. *Suppose that $\Omega \equiv [\underline{\omega}, \overline{\omega}]$ and that assumptions (2) and (3) hold. Further, assume that for all $\omega \in \Omega$, $\mathbb{E}\left\{\boldsymbol{y}\frac{\frac{\partial f(\boldsymbol{y}|\omega)}{\partial \omega}}{f(\boldsymbol{y}|\omega)}|\omega\right\} < \infty$. Then, the revenue maximizing mechanism is* *characterized by a menu of debt securities given by*

$$s^{*}(y|\omega)=\min\{y,D_{*}(\omega)\}$$

where $D_{*}(\omega)$ is defined as the unique solution of

$$\int_{D_{*}(\omega)}^{\infty}\left\{1-\delta-\frac{1-\Phi(\omega)}{\phi(\omega)}\frac{\frac{\partial}{\partial\omega}f\left(y|\omega\right)}{f\left(y|\omega\right)}\right\}\mathrm{d}F\left(y|\omega\right)\;\;=\;\;0. \tag{24}$$

Proof. First note that, because $\mathbb{E}\left\{\boldsymbol{y}\frac{\frac{\partial f(\boldsymbol{y}|\omega)}{\partial\omega}}{f(\boldsymbol{y}|\omega)}\right\}<\infty$, for any feasible mechanism $\mathcal{M}=\left\{s\left[\omega\right],p\left(\omega\right)\right\}_{\omega\in\Omega}$, there exists an integrable function $b:\Omega\rightarrow\mathbb{R}$, satisfying

$$\left|\frac{\partial}{\partial\omega}U_{\mathcal{M}}\left(\tilde{\omega},\omega\right)\right|=\left|\mathbb{E}\left\{s\left(\boldsymbol{y}|\tilde{\omega}\right)\frac{\frac{\partial f(\boldsymbol{y}|\omega)}{\partial\omega}}{f\left(\boldsymbol{y}|\omega\right)}\right\}\right|\leq b\left(\omega\right),\;\forall\tilde{\omega},\omega.$$

The arguments in Milgrom and Segal [2002] then imply that, because $U_{\mathcal{M}}\left(\hat{\omega};\omega\right)$ is absolutely continuous in ω, $U_{\mathcal{M}}\left(\omega,\omega\right)$ is absolutely continuous. Furthermore,

$$U_{\mathcal{M}}\left(\omega,\omega\right)-U_{\mathcal{M}}\left(\underline{\omega},\underline{\omega}\right)=\int_{\underline{\omega}}^{\omega}\left(\int_{0}^{\infty}s(y|\tilde{\omega})\frac{\partial}{\partial\omega}f(y|\tilde{\omega})\mathrm{d}y\right)\mathrm{d}\tilde{\omega},\text{ for }\Phi-\text{almost all }\omega.$$

This means that, for any incentive compatible mechanism, the price paid by type ω is given by

$$\begin{aligned}p\left(\omega\right)&=\mathbb{E}\left\{s\left(\boldsymbol{y}|\omega\right)\right\}-U_{\mathcal{M}}\left(\omega,\omega\right)\\&=\int_{\Omega}\left\{\int_{0}^{\infty}s(y|\omega)\mathrm{d}F(y|\omega)-\left(\int_{\underline{\omega}}^{\omega}\left(\int_{0}^{\infty}s(y|\tilde{\omega})\frac{\partial}{\partial\omega}f(y|\tilde{\omega})\mathrm{d}y\right)\mathrm{d}\tilde{\omega}\right)\right\}\mathrm{d}\Phi\left(\omega\right)-U_{\mathcal{M}}\left(\underline{\omega},\underline{\omega}\right)\\&=\int_{\Omega}\int_{0}^{\infty}\left(s(y|\omega)\left(1-\left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right)\frac{\frac{\partial}{\partial\omega}f(y|\omega)}{f(y|\omega)}\right)\right)\mathrm{d}F(y|\omega)\mathrm{d}\Phi\left(\omega\right)-U_{\mathcal{M}}\left(\underline{\omega},\underline{\omega}\right),\end{aligned}$$

and, therefore, we can rewrite the issuer's payoff, $\mathbb{E}\left\{p\left(\boldsymbol{\omega}\right)+\delta\left(\boldsymbol{y}-s\left(\boldsymbol{y}|\boldsymbol{\omega}\right)\right)\right\}$, as

$$\begin{aligned}&\delta\mathbb{E}\left\{\boldsymbol{y}\right\}+\int_{\Omega}\left(\mathbb{E}\left\{s\left(\boldsymbol{y}|\omega\right)\right\}-U_{\mathcal{M}}\left(\omega,\omega\right)-\delta\mathbb{E}\left\{s\left(\boldsymbol{y}|\omega\right)\right\}\right)\mathrm{d}\Phi\left(\omega\right)-U_{\mathcal{M}}\left(\underline{\omega},\underline{\omega}\right)\\=\;&\delta\mathbb{E}\left\{\boldsymbol{y}\right\}+\int_{\Omega}\left\{\int_{0}^{\infty}\left(1-\delta\right)s(y|\omega)\mathrm{d}F(y|\omega)-\int_{\underline{\omega}}^{\omega}\left(\int_{0}^{\infty}s(y|\tilde{\omega})\frac{\partial}{\partial\omega}f(y|\tilde{\omega})\mathrm{d}y\right)\mathrm{d}\tilde{\omega}\right\}\mathrm{d}\Phi\left(\omega\right)-U_{\mathcal{M}}\left(\underline{\omega},\underline{\omega}\right)\\=\;&\delta\mathbb{E}\left\{\boldsymbol{y}\right\}+\int_{\Omega}\int_{0}^{\infty}\left(s(y|\omega)\left(1-\delta-\left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right)\frac{\frac{\partial}{\partial\omega}f(y|\omega)}{f(y|\omega)}\right)\right)\mathrm{d}F(y|\omega)\mathrm{d}\Phi\left(\omega\right)-U_{\mathcal{M}}\left(\underline{\omega},\underline{\omega}\right).\end{aligned}$$

where the second equality obtains from integration by parts.

The next result indetifies a sufficient condition for a mechanism to be incentive compatible.

Proposition 6. *Consider an arbitrary mechanism* $\mathcal{M} = \{s(\cdot|\omega), p(\omega)\}_{\omega\in\Omega}$ *satisfying*

$$\left(\int_0^\infty (s(y|\omega) - s(y|\hat{\omega})) \frac{\partial}{\partial\omega} f(y|\omega)\, dy\right) \cdot (\omega - \hat{\omega}) \geq 0,\ \forall \omega, \hat{\omega} \in \Omega. \tag{25}$$

Then, $\mathcal{M}$ *is incentive compatible.*

Proof. Consider an arbitrary mechanism $\mathcal{M} = \{s(\cdot|\omega), p(\omega)\}_{\omega\in\Omega}$ satisfying (25). For any $\omega, \hat{\omega} \in \Omega$, let

$$Q(\hat{\omega}, \omega) \equiv U_{\mathcal{M}}(\omega, \omega) - U_{\mathcal{M}}(\hat{\omega}, \omega).$$

Note that, for any $\hat{\omega}$, $Q(\hat{\omega}, \cdot)$ is absolutely continuous. Moreover, $Q(\hat{\omega}, \hat{\omega}) = 0$ for all $\hat{\omega} \in \Omega$. This implies that, for any $\omega, \hat{\omega} \in \Omega$,

$$\begin{aligned}
Q(\hat{\omega}, \omega) &= Q(\hat{\omega}, \omega) - Q(\hat{\omega}, \hat{\omega}) \\
&= \int_{\hat{\omega}}^{\omega} \frac{\partial Q(\hat{\omega}, z)}{\partial\omega} \mathrm{d}z \\
&= \int_{\hat{\omega}}^{\omega} \left\{ \frac{\mathrm{d}}{\mathrm{d}\omega} U_{\mathcal{M}}(\omega, \omega)\Big|_{\omega=z} - \frac{\partial}{\partial\omega} U_{\mathcal{M}}(\hat{\omega}, \omega)\Big|_{\omega=z} \right\} \mathrm{d}z \\
&= \int_{\hat{\omega}}^{\omega} \left\{ \int_0^\infty (s(y|z) - s(y|\hat{\omega})) \frac{\partial}{\partial\omega} f(y|z)\mathrm{d}y \right\} \mathrm{d}z \\
&\geq 0,
\end{aligned}$$

where the inequality follows from (25). We thus conclude that $U_{\mathcal{M}}(\omega, \omega) \geq U_{\mathcal{M}}(\hat{\omega}, \omega)$, for any $\omega, \hat{\omega} \in \Omega$. □

The strategy of the proof consists in ignoring constraint (25) and finding, for each ω, the security $s^*(\cdot|\omega)$ which pointwise maximizes the issuer's payoff. We then show that, when (2) and (3) hold, the securities $\{s^*(\cdot|\omega)\}_{\omega\in\Omega}$ satisfy constraint (25).

Lemma 7. *Consider an arbitrary incentive compatible mechanism* $\mathcal{M} = \{s(\cdot|\omega), p(\omega)\}_{\omega\in\Omega}$. *Suppose that there exists a set* $\Omega^{Non\text{-}debt} \subseteq \Omega$ *with positive* $\Phi-$*measure, such that, for any* $\omega \in \Omega^{Non\text{-}debt}$, $s(\cdot|\omega)$ *is not a debt security. There exists a set of debt securities* $\left\{s^{\#\#}(\cdot|\omega)\right\}_{\omega\in\Omega^{Non\text{-}debt}}$ *so that the mechanism* $\mathcal{M}^{\#\#}$ *constructed from* $\mathcal{M}$ *by replacing* $s(\cdot|\omega)$ *for* $s^{\#\#}(\cdot|\omega)$ *for all* $\omega \in \Omega^{Non\text{-}debt}$, *dominates* $\mathcal{M}$ *for the issuer.*

Proof. For any $\omega \in \Omega^{\text{Non-debt}}$, let

$$k(y, \omega; \delta) \equiv 1 - \delta - \left(\frac{1 - \Phi(\omega)}{\phi(\omega)} \right) \frac{\frac{\partial}{\partial \omega} f\left(y|\omega\right)}{f\left(y|\omega\right)}.$$

The issuer's payoff is thus given by

$$\delta \mathbb{E}\left\{\boldsymbol{y}\right\} + \int_{\Omega} \int_{0}^{\infty} s(y|\omega) k(y, \omega) \mathrm{d}F(y|\omega) \mathrm{d}\Phi\left(\omega\right) - U_{\mathcal{M}}\left(\underline{\omega}, \underline{\omega}\right).$$

The fact that $f\left(y|\omega\right)$ satisfies MLRP implies that $k(\cdot, \omega; \delta)$ satisfies the SSCFA property. Let $y_0(\omega; \delta)$ be the unique solution to $k(y_0(\omega; \delta), \omega) = 0$. From the definition of $D_*(\omega; \delta)$ in 24, we must necessarily have that $y_0(\omega; \delta) > D_*(\omega; \delta)$, for all $\omega \in \Omega$. The constraint that all securities in $\mathcal{S}$ are nondecreasing, together with the fact that $k(y, \omega; \delta) < 0$ for all $y > y_0\left(\omega; \delta\right)$, jointly imply that, any security $\tilde{s}(\cdot|\omega)$ in $\mathcal{S}$ which fails to be constant to the right of $y_0\left(\omega; \delta\right)$ is dominated by the security

$$s^{\#}(y|\omega) \equiv \tilde{s}(y|\omega) 1\{y \leq y_0(\omega; \delta)\} + \tilde{s}(y_0(\omega; \delta)|\omega) 1\{y > y_0(\omega)\}.$$

Finally, the fact that $k(y|\omega; \delta) > 0$ for all $y < y_0\left(\omega; \delta\right)$ implies that any security $\bar{s}(y|\omega) \in \mathcal{S}$ satisfying $\bar{s}(y|\omega) = d$ for all $y \geq y_0(\omega; \delta)$ is weakly dominated by the security

$$\begin{aligned} s^{\#\#}(y|\omega) &= \min\left\{y, d\right\} \cdot 1\{y < y_0(\omega; \delta)\} + d \cdot 1\{y \geq y_0(\omega; \delta)\}. \\ &= \min\left\{y, d\right\}. \end{aligned}$$

This proves the lemma.□

Lemma 7 implies that, for any ω, debt securities pointwise maximize issuer's payoff. The issuer then chooses $\left\{D\left(\omega\right)\right\}_{\omega \in \Omega}$ to maximize

$$\int_{\Omega} \left\{ \int_{0}^{D(\omega)} y k(y, \omega; \delta) \mathrm{d}F(y|\omega) + D\left(\omega\right) \int_{D(\omega)}^{\infty} k(y, \omega; \delta) \mathrm{d}F(y|\omega) \right\} \mathrm{d}\Phi\left(\omega\right).$$

Note that when (2) and (3) hold, the function

$$\chi\left(\omega, D\left(\omega\right)\right) \equiv \int_{0}^{D(\omega)} y k(y, \omega; \delta) \mathrm{d}F(y|\omega) + D\left(\omega\right) \int_{D(\omega)}^{\infty} k(y, \omega; \delta) \mathrm{d}F(y|\omega)$$

is supermodular. Indeed,

$$\begin{aligned}
\frac{\partial^2}{\partial\omega\partial D(\omega)}\chi(\omega, D(\omega)) &= \frac{\partial}{\partial\omega}\int_{D(\omega)}^{\infty} k(y,\omega;\delta)\mathrm{d}F(y|\omega) \\
&= \frac{\partial}{\partial\omega}\left\{(1-\delta)(1-F(D(\omega)|\omega)) - \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\frac{\partial}{\partial\omega}(1-F(D(\omega)|\omega))\right\} \\
&= (1-\delta)\underbrace{\frac{\partial}{\partial\omega}(1-F(D(\omega)|\omega))}_{>0 \text{ (FOSD)}} - \underbrace{\frac{\partial}{\partial\omega}\left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)}_{<0 \text{ (Assumption2)}}\underbrace{\frac{\partial}{\partial\omega}(1-F(D(\omega)|\omega))}_{>0 \text{ (FOSD)}} \\
&\quad - \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\cdot\underbrace{\frac{\partial^2}{\partial\omega^2}(1-F(D(\omega)|\omega))}_{<0 \text{ (Assumption3)}} \\
&> 0.
\end{aligned}$$

Topkis Theorem then implies that the value of $D(\omega)$ that maximizes (pointwise) $\chi(\omega, D(\omega))$, $D_*(\omega;\delta)$, must be increasing in ω. This further implies that the constraint (25) is satisfied, and therefore the set of pointwise optimal securities is feasible.

This completes the proof of the Theorem. □

Appendix C: Additional Proofs

Proof of Example 1. We start by showing that θ orders the experiments in the Lehmann sense.

Claim 1. For any $\theta'' > \theta'$, $F^{\theta''} \succeq_{\text{Lehmann}} F^{\theta'}$.

Proof. For any $\theta \in [0,1]$, let $F^{\theta}_{\boldsymbol{\omega}}\left(\omega|\boldsymbol{y}^{\theta} > z\right) \equiv \mathbb{P}\left\{\boldsymbol{\omega} \leq \omega||\boldsymbol{y}^{\theta} > z\right\}$. Following Theorem 1 in Athey and Levin [2018], it is enough to prove that, for any $u : [0,1] \to \mathbb{R}$ satisfying SCFB,

$$\int_0^1 u(\omega)\,\mathrm{d}F^{\theta'}_{\boldsymbol{\omega}}\left(\omega|\boldsymbol{y}^{\theta'} > z\right) \geq 0 \Rightarrow \int_0^1 u(\omega)\,\mathrm{d}F^{\theta''}_{\boldsymbol{\omega}}\left(\omega|\boldsymbol{y}^{\theta''} > z\right) \geq 0. \tag{26}$$

First, note that, for any $\theta \in [0,1]$, and any $z \in [0,1]$,

$$
\begin{aligned}
F^{\theta}_{\boldsymbol{\omega}}\left(\omega|\boldsymbol{y}^{\theta}>z\right) &= \frac{\mathbb{P}\left\{\boldsymbol{y}^{\theta}>z|\boldsymbol{\omega}\leq\omega\right\}\omega}{\int_0^1\left(1-F^{\theta}\left(z|\tilde{\omega}\right)\right)\mathrm{d}\tilde{\omega}} \\
&= \frac{\omega\int_0^{\omega}\left(1-F^{\theta}\left(z|\tilde{\omega}\right)\right)\mathrm{d}\tilde{\omega}}{\int_0^1\left(1-F^{\theta}\left(z|\tilde{\omega}\right)\right)\mathrm{d}\tilde{\omega}} \\
&= \frac{\omega\left(\omega-\int_0^{\omega}\left(\theta\cdot 1\left\{\tilde{\omega}\leq z\right\}+\left(1-\theta\right)z\right)\mathrm{d}\tilde{\omega}\right)}{1-\int_0^1\left(\theta\cdot 1\left\{\tilde{\omega}\leq z\right\}+\left(1-\theta\right)z\right)\mathrm{d}\tilde{\omega}} \\
&= \frac{\omega\left(\omega-\left(1-\theta\right)z\omega-\int_0^{\omega}\left(\theta\cdot 1\left\{\tilde{\omega}\leq z\right\}\right)\mathrm{d}\tilde{\omega}\right)}{1-z} \\
&= \frac{\omega\left(\omega-\theta\min\left\{\omega,z\right\}-\left(1-\theta\right)z\omega\right)}{1-z} \\
&= \begin{cases}\omega^2\left(1-\theta\right) & \text{if } \omega<z \\ \frac{\omega\left(\omega-\left(1-\theta\right)z\omega-\theta z\right)}{1-z} & \text{if } \omega\geq z\end{cases}
\end{aligned}
$$

This then implies that, for any $\omega\neq z$,

$$
f^{\theta}_{\boldsymbol{\omega}}\left(\omega|\boldsymbol{y}^{\theta}>z\right)=\begin{cases}2\omega\left(1-\theta\right) & \text{if } \omega<z \\ \frac{2\omega\left(1-\left(1-\theta\right)z\right)-\theta z}{1-z} & \text{if } \omega>z.\end{cases}
$$

We further note that $F^{\theta}_{\boldsymbol{\omega}}\left(\omega|\boldsymbol{y}^{\theta}>z\right)$ is continuous at $\omega=z$ and hence absolutely continuous over $[0,1]$.

Finally, we note that

$$
\frac{\frac{\mathrm{d}}{\mathrm{d}\omega}f^{\theta}_{\boldsymbol{\omega}}\left(\omega|\boldsymbol{y}^{\theta}>z\right)}{f^{\theta}_{\boldsymbol{\omega}}\left(\omega|\boldsymbol{y}^{\theta}>z\right)}=\begin{cases}\frac{1}{\omega} & \text{if } \omega<z \\ \frac{2\left(1-\left(1-\theta\right)z\right)}{2\omega\left(1-\left(1-\theta\right)z\right)-\theta z} & \text{if } \omega>z\end{cases}
$$

is nondecreasing in θ for any $\omega\in[0,1)$. This implies that the density $f^{\theta}_{\boldsymbol{\omega}}\left(\omega|\boldsymbol{y}^{\theta}>z\right)$ is log-supermodular in (θ,ω). Lemma 1 then implies that for any $u:[0,1]\to\mathbb{R}$ satisfying SCFB, (26) is satisfied. This proves the claim. *q.e.d.*

Claim 2. Suppose the issuer is restricted to use linear securities,

$$
s\in\mathcal{S}^E\equiv\left\{s\in\mathcal{S}:\exists\alpha>0,s\left(y\right)=\alpha y,\forall y\in[0,1]\right\}.
$$

Then, the (restricted) optimal mechanism is characterized by

$$\alpha_{\theta}^{*}(\omega)=1\left\{\omega\geq\omega_{\theta}^{*}\equiv\max\left\{\frac{3\theta-1}{4\theta},0\right\}\right\}.$$

Proof. We consider mechanisms of the form $\mathcal{M}_E^{\theta}=\left\{s_E^{\theta}[\omega],p_E^{\theta}(\omega)\right\}_{\omega\in[0,1]}$, where for all $\omega\in[0,1]$, $s_E^{\theta}(y|\omega)=\alpha^{\theta}(\omega)\,y$ and $p_E^{\theta}(\omega)\in\mathbb{R}$. Note that for any $\omega\in[0,1]$,

$$\mathbb{E}\left\{\boldsymbol{y}^{\theta}|\boldsymbol{\omega}=\omega\right\}=\underbrace{\theta\omega+\frac{1-\theta}{2}}_{\equiv h^{\theta}(\omega)}.$$

Furthermore, $h^{\theta}(\boldsymbol{\omega})=\mathbb{E}\left\{\boldsymbol{y}^{\theta}|\boldsymbol{\omega}\right\}\sim U\left[\frac{1-\theta}{2},\frac{1+\theta}{2}\right]$.

Next, for any $\omega,\tilde{\omega}\in[0,1]$ let

$$U_{\mathcal{M}_E^{\theta}}(\tilde{\omega};\omega)=\alpha^{\theta}(\tilde{\omega})\,h^{\theta}(\omega)-p_E^{\theta}(\tilde{\omega})$$

be the liquidity supplier's payoff when his true type is ω and he chooses to report $\tilde{\omega}$. The liquidity supplier's IC constraint is then given by

$$U_{\mathcal{M}_E^{\theta}}(\omega;\omega)=\max_{\tilde{\omega}}U_{\mathcal{M}_E^{\theta}}(\tilde{\omega};\omega).$$

Using the envelope theorem, we obtain that

$$\frac{\mathrm{d}}{\mathrm{d}\omega}U_{\mathcal{M}_E^{\theta}}(\omega;\omega)=\alpha^{\theta}(\omega)\frac{\mathrm{d}h^{\theta}(\omega)}{\mathrm{d}\omega},\ \forall\omega\in[0,1].$$

This further implies that

$$U_{\mathcal{M}_E^{\theta}}(\omega;\omega)=\int_0^{\omega}\alpha^{\theta}(\tilde{\omega})\frac{\mathrm{d}h^{\theta}(\tilde{\omega})}{\mathrm{d}\omega}\mathrm{d}\tilde{\omega}.$$

We conclude that

$$p_E^{\theta}(\omega)=\alpha^{\theta}(\omega)\,h^{\theta}(\omega)-\int_0^{\omega}\alpha^{\theta}(\tilde{\omega})\frac{\mathrm{d}h^{\theta}(\tilde{\omega})}{\mathrm{d}\omega}\mathrm{d}\tilde{\omega},$$

and therefore

$$\begin{aligned}\mathbb{E}\left\{p_E^\theta(\boldsymbol{\omega})\right\} &= \int_0^1 \left\{\alpha^\theta(\omega) h^\theta(\omega) - \int_0^\omega \alpha^\theta(\tilde{\omega}) \frac{\mathrm{d}h^\theta(\tilde{\omega})}{\mathrm{d}\omega}\mathrm{d}\tilde{\omega}\right\}\mathrm{d}\omega \\ &= \int_0^1 \left\{\alpha^\theta(\omega)\left(h^\theta(\omega) - (1-\omega)\frac{\mathrm{d}h^\theta(\omega)}{\mathrm{d}\omega}\right)\right\}\mathrm{d}\omega \\ &= \int_0^1 \left\{\alpha^\theta(\omega)\left(\theta\omega + \frac{1-\theta}{2} - (1-\omega)\theta\right)\right\}\mathrm{d}\omega \\ &= \int_0^1 \left\{\alpha^\theta(\omega)\left(\theta\left(\frac{4\omega-3}{2}\right) + \frac{1}{2}\right)\right\}\mathrm{d}\omega.\end{aligned}$$

The issuer's can then be written as

$$\begin{aligned}\max_{\left\{\alpha^\theta(\omega)\right\}_{\omega\in[0,1]}} \quad & \mathbb{E}\left\{p_E^\theta(\boldsymbol{\omega})\right\} = \int_0^1 \left\{\alpha^\theta(\omega)\left(\theta\left(\frac{4\omega-3}{2}\right) + \frac{1}{2}\right)\right\}\mathrm{d}\omega \\ \text{s.t.} \quad & \alpha^\theta(\cdot) \text{ nondecreasing.}\end{aligned}$$

The issuer therefore optimally sets

$$\alpha_*^\theta(\omega) = 1\left\{\omega \geq \omega_\theta^* \equiv \max\left\{\frac{3\theta-1}{4\theta}, 0\right\}\right\},$$

as claimed. *q.e.d.*

Claim 3. The optimal mechanism is characterized as follows. For any $\theta \in \left(\frac{1}{7}, 1\right)$, $D_\theta^*(\omega) = \omega$ for all $\omega \in [0,1]$. In turn, for any $\theta \in \left(0, \frac{1}{7}\right)$, $D_\theta^*(\omega) = 1$ for all $\omega \in [0,1]$.

Proof. Fix any $\theta \in (0,1)$ and let $\mathcal{M}_*^\theta = \left\{s^\theta \equiv \min\left\{y, D_*^\theta(\omega)\right\}, p_*^\theta(\omega)\right\}_{\omega\in\in[0,1]}$ represent the optimal menu of debt contracts (existence follows from the derivation below). Next, for any $\omega, \tilde{\omega} \in [0,1]$ let

$$\begin{aligned}U_{\mathcal{M}_*^\theta}^\theta(\tilde{\omega};\omega) &\equiv \mathbb{E}\left\{\min\left\{\boldsymbol{y}, D_*^\theta(\tilde{\omega})\right\}|\omega\right\} - p_*^\theta(\tilde{\omega}). \\ &= (1-\theta)\left(D_*^\theta(\tilde{\omega}) - \frac{D_*^\theta(\tilde{\omega})^2}{2}\right) + \theta\min\left\{\omega, D_*^\theta(\tilde{\omega})\right\} - p_*^\theta(\tilde{\omega})\end{aligned}$$

The fact that, downward incentive compatibility constraints bind implies that

$$\frac{\mathrm{d}}{\mathrm{d}\omega}U_{\mathcal{M}_*^\theta}^\theta(\omega;\omega) = \theta \cdot 1\left\{D(\omega) > \omega\right\},$$

and, therefore,

$$U^{\theta}_{\mathcal{M}^{\theta}_{*}}(\omega;\omega) = U^{\theta}_{\mathcal{M}^{\theta}_{*}}(0;0) + \theta\int_0^{\omega} 1\{D(\tilde{\omega}) > \tilde{\omega}\}\,\mathrm{d}\tilde{\omega}.$$

As a result, we obtain that the issuer's revenue is given by

$$\begin{aligned}\mathbb{E}\left\{p^{\theta}(\boldsymbol{\omega})\right\} &= \int_0^1 \left\{(1-\theta)\left(D^{\theta}_{*}(\omega) - \frac{D^{\theta}_{*}(\omega)^2}{2}\right) + \theta\left(\min\{\omega, D^{\theta}_{*}(\omega)\} - \int_0^{\omega} 1\{D^{\theta}_{*}(\tilde{\omega}) > \tilde{\omega}\}\mathrm{d}\tilde{\omega}\right)\right\}\mathrm{d}\omega \\ &= \int_0^1 \left\{(1-\theta)\left(D^{\theta}_{*}(\omega) - \frac{D^{\theta}_{*}(\omega)^2}{2}\right) + \theta\min\{\omega, D^{\theta}_{*}(\omega)\} - \theta(1-\omega)1\{D^{\theta}_{*}(\omega) > \omega\}\right\}\mathrm{d}\omega.\end{aligned}$$

Observe first that $D^{\theta}_{*}(\omega) \geq \omega$ for all ω. Indeed, for any ω, the first term is strictly increasing in $D^{\theta}_{*}(\omega)$, whereas the second term is also strictly increasing for any $D^{\theta}_{*}(\omega) < \omega$. The last term, in contrast, is constant in $D^{\theta}_{*}(\omega)$ everywhere except at ω where it suffers a discontinuous jump. Any menu for which $D^{\theta}_{*}(\omega) < \omega$ over a set with positive measure $\Omega^{-} \equiv \left\{\omega \in [0,1] : D^{\theta}_{*}(\omega) < \omega\right\}$ can then be strictly dominated by slightly increasing the value of $D^{\theta}_{*}(\cdot)$ for $\omega \in (\sup\Omega^{-} - \epsilon, \sup\Omega^{-})$, for $\epsilon > 0$ small.

Next, note that for any ω for which $D^{\theta}_{*}(\omega) > \omega$, it is (pointwise) optimal to set $D^{\theta}_{*}(\omega) = 1$, as both the second and third terms are invariants to increments in $D^{\theta}_{*}(\omega)$, whereas the first term is strictly increasing in $D^{\theta}_{*}(\omega)$.

We conclude that the optimal mechanism must take the form

$$D^{\theta}_{*}(\omega) = \begin{cases} \omega & , \text{for } \omega < x \\ 1 & , \text{for } \omega \geq x \end{cases}$$

for some $x \in [0,1]$. We thus optimize $\mathbb{E}\left\{p^{\theta}_{*}(\boldsymbol{\omega})\right\}$ by changing the value of x. For any such a menu, we have

$$\mathbb{E}\left\{p^{\theta}_{*}(\boldsymbol{\omega})\right\} = (1-\theta)\left(\int_0^x \left(\omega - \frac{\omega^2}{2}\right)\mathrm{d}\omega + \frac{1-x}{2}\right) + \theta\int_0^1 \left(\omega - (1-\omega)1\{\omega > x\}\right)\mathrm{d}\omega.$$

This further implies that

$$\frac{\mathrm{d}}{\mathrm{d}x}\mathbb{E}\left\{p^{\theta}_{*}(\boldsymbol{\omega})\right\} = \frac{(1-x)\left((1-\theta)x - (1-3\theta)\right)}{1-\theta}.$$

We finally note that, for any $\theta \in \left[\frac{1}{3}, 1\right)$, $\frac{\mathrm{d}}{\mathrm{d}x}\mathbb{E}\left\{p^{\theta}_{*}(\boldsymbol{\omega})\right\} \geq 0$ for all $x \in [0,1]$. We conclude that in that case, it is optimal to choose $x = 1$, and therefore $D^{\theta}_{*}(\omega) = \omega$ for all $\omega \in [0,1]$. In contrast, when $\theta < 1/3$, $\mathbb{E}\left\{p^{\theta}_{*}(\boldsymbol{\omega})\right\}$ is quasi-convex in x and the optimal choice is found at the corners. We thus need to compare the value of $\mathbb{E}\left\{p^{\theta}_{*}(\boldsymbol{\omega})\right\}$ at $x = 0$ and $x = 1$. We find that

$$\begin{aligned}\mathbb{E}\left\{p_*^\theta(\boldsymbol{\omega})\right\}\Big|_{x=1} &= (1-\theta)\left(\int_0^1\left(\omega-\frac{\omega^2}{2}\right)\mathrm{d}\omega\right)+\theta\int_0^1\omega\mathrm{d}\omega \\ &= \frac{2+\theta}{6},\end{aligned}$$

whereas

$$\begin{aligned}\mathbb{E}\left\{p^\theta(\boldsymbol{\omega})\right\}\Big|_{x=0} &= \frac{1-\theta}{2}+\theta\int_0^1(2\omega-1)\,\mathrm{d}\omega, \\ &= \frac{1-\theta}{2}.\end{aligned}$$

We conclude that for any $\theta\in\left(\frac{1}{7},\frac{1}{3}\right)$, it is optimal to set $x=1$, and then $D_*^\theta(\omega)=\omega$ for all $\omega\in[0,1]$, whereas, for any $\theta\in\left(0,\frac{1}{7}\right)$, it is optimal to set $x=0$, and then $D_*^\theta(\omega)=1$ for all $\omega\in[0,1]$. *q.e.d.*

Proof of Proposition 2. Claim (i) follows from Proposition 2 in Jewitt [2007] which states that, an experiment F'' Lehmann-dominates another experiment F', if and only if, for any arbitrary prior distribution Φ, the induced joint distributions $\mathbf{F}''_\Phi$ and $\mathbf{F}'_\Phi$ are ranked in the positive quadrant dependence (PQD) order.[28] For random vectors of dimension $N=2$, the PQD order, in turn, is equivalent to the supermodular order (Tchen [1980]). Claim (ii) is standard (see, e.g., Müller and Stoyan [2002]) and follows from the fact that the domination in the supermodular order implies a higher degree of interdependence. Claim (iii) follows from noting that, for any nondecreasing function $u(\cdot)$, and any $z\in[0,1]$, the function $\mathbb{I}\{\Phi(\omega)\geq z\}\,u(y)$ is supermodular in (y,ω) and, therefore,

$$\int_{\mathbb{R}_+}\int_\Omega\mathbb{I}\{\Phi(\omega)\geq z\}\,u(y)\,\mathrm{d}\mathbf{F}''_\Phi(y,\omega)\geq\int_{\mathbb{R}_+}\int_\Omega\mathbb{I}\{\Phi(\omega)\geq z\}\,u(y)\,\mathrm{d}\mathbf{F}'_\Phi(y,\omega).$$

This means that, for any nondecreasing function $u(\cdot)$,

$$\mathbb{E}_{\mathbf{F}''_\Phi}(u(\boldsymbol{y})\,|\Phi(\boldsymbol{\omega})\geq z)\geq\mathbb{E}_{\mathbf{F}'_\Phi}(u(\boldsymbol{y})\,|\Phi(\boldsymbol{\omega})\geq z),$$

and, therefore ,

$$\mathbf{F}''_\Phi(y|\Phi(\boldsymbol{\omega})\geq z)\succeq_{\mathrm{FOSD}}\mathbf{F}'_\Phi(y|\Phi(\boldsymbol{\omega})\geq z),\ \forall z\in[0,1].$$

[28] A distribution $P\in\Delta\mathbb{R}^N$ dominates $Q\in\Delta\mathbb{R}^N$ in the PQD order, if $P(z_1,...,z_N)\leq Q(z_1,...,z_N)$, $\forall(z_1,...,z_N)\in\mathbb{R}^N$. See, e.g., Shaked and Shanthikumar [2007].

In other words, $\mathbf{F}''_\Phi$ dominates $\mathbf{F}'_\Phi$ in the Monotone Information Order for Nondecreasing objective functions (MIO-ND) sense (see Athey and Levin [2018]). Theorem 1 in Ganuza and Penalva [2010] then implies that $\mathbb{E}_{\mathrm{F}''_\Phi}(\boldsymbol{y}|\boldsymbol{\omega}) \succeq_{\mathrm{cvx}} \mathbb{E}_{\mathrm{F}'_\Phi}(\boldsymbol{y}|\boldsymbol{\omega})$. □

Proof of Lemma 5. Note that the issuer's revenue is given by

$$\Pi_*(F) = \mathbb{E}\{p_*(\boldsymbol{\omega}; F) + \delta(\boldsymbol{y} - s_*(\boldsymbol{y}|\boldsymbol{\omega}))\} = \delta\mathbb{E}\{\boldsymbol{y}\} + \mathbb{E}\{p_*(\boldsymbol{\omega}; F) - \delta s_*(\boldsymbol{y}|\boldsymbol{\omega})\}.$$

The assumptions in the model guarantee that as the accuracy of the private signal changes, the marginal distribution of $\boldsymbol{y}$ remains unchanged. Thus, the issuer's revenue dependence on F is fully determined by $\Pi_*(F) - \delta\mathbb{E}\{\boldsymbol{y}\} = \mathbb{E}\{p_*(\boldsymbol{\omega}; F) - \delta s_*(\boldsymbol{y}|\boldsymbol{\omega})\}$, where

$$\begin{aligned}
&\Pi_*(F) - \delta\mathbb{E}\{\boldsymbol{y}\} \\
=\ & \int_\Omega \left\{ \int_{\mathbb{R}_+} \min\{y, D_*(\omega; F)\} \left(1 - \delta - \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\left(\frac{\frac{\partial f(y|\omega)}{\partial \omega}}{f(y|\omega)}\right)\right) \mathrm{d}F(y|\omega) \right\} \mathrm{d}\Phi(\omega) \\
=\ & \int_\Omega \left\{ \int_0^{D_*(\omega;F)} y \left(1 - \delta - \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\left(\frac{\frac{\partial f(y|\omega)}{\partial \omega}}{f(y|\omega)}\right)\right) \mathrm{d}F(y|\omega) \right\} \mathrm{d}\Phi(\omega) \\
=\ & \int_\Omega \left\{ \int_0^{D_*(\omega;F)} (1-\delta)\left(F(D_*(\omega;F)|\omega) - F(y|\omega)\right)\mathrm{d}y \right. \\
& \left. - \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right) \int_0^{D_*(\omega;F)} \frac{\partial}{\partial\omega}\left(F(D_*(\omega;F)|\omega) - F(y|\omega)\right)\mathrm{d}y \right\} \mathrm{d}\Phi(\omega) \\
=\ & \int_\Omega \int_0^{D_*(\omega;F)} (1 - F(y|\omega)) \left\{ 1 - \delta - \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right) \frac{\frac{\partial}{\partial\omega}(1 - F(y|\omega))}{1 - F(y|\omega)} \right\} \mathrm{d}y\mathrm{d}\Phi(\omega),
\end{aligned}$$

where the second equality obtains from the definition of $D_*(\omega; F)$, the third equality follows from applying integration by parts, and the fourth equality obtains from rearranging terms and using the definition of $D_*(\omega; F)$, which implies that

$$(1-\delta)(1 - F(D_*(\omega;F)|\omega)) - \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\frac{\partial}{\partial\omega}(1 - F(D_*(\omega;F)|\omega)) = 0.$$

This completes the proof of the lemma. □

Lemma 8. *Suppose that $\delta = 0$ (strong liquidity constraints) and that, for all y, the function*

$$\zeta(y, \omega; F) \equiv (1 - \Phi(\omega))(1 - F(y|\omega))$$

has increasing differences in ω and accuracy. Then, for any $F'' \succeq_{Lehmann} F'$, $\mathbb{E}(p_*(\boldsymbol{\omega}; F'')) \leq$

$\mathbb{E}\left(p_*\left(\boldsymbol{\omega}; F'\right)\right)$.

Proof. From the derivation above, we know that

$$\begin{aligned}\mathbb{E}\left(p_*\left(\boldsymbol{\omega}; F\right)\right) &= \int_\Omega \int_0^{D_*(\omega;F)} \left\{1 - F\left(y|\omega\right) - \left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right) \frac{\partial}{\partial \omega}\left(1 - F\left(y|\omega\right)\right)\right\} \mathrm{d}y \mathrm{d}\Phi\left(\omega\right). \\ &= \int_\Omega \int_0^{D_*(\omega;F)} -\frac{\partial}{\partial \omega} \zeta\left(y, \omega; F\right) \mathrm{d}y \mathrm{d}\omega\end{aligned}$$

Assuming that $\zeta\left(y, \omega; F\right)$ has increasing differences in ω and accuracy is equivalent to stating that effect (c) dominates effect (b). When $\zeta\left(y, \omega; F\right)$ has this property, the amount of funds raised $\mathbb{E}\left(p_*\left(\boldsymbol{\omega}; F\right)\right)$ decreases with the accuracy of F.

Intuitively, for each type ω, $-\frac{\partial}{\partial \omega}\partial \zeta\left(y|\omega, F\right)$ represents the issuer's marginal incentive to increase the face value of type ω's debt contract accounting for the information rents that have to be given up to all types above ω. Indeed, from equation (7), we have

$$\frac{\partial}{\partial D\left(\omega\right)} \mathbb{E}\left(p\left(\boldsymbol{\omega}\right); F\right) = -\frac{\partial \zeta\left(y, \omega; F\right)}{\partial \omega}.$$

The increasing difference assumption then guarantees that as the liquidity supplier's private signal becomes more informative, the issuer's virtual valuation (that is, gains from trade minus information rents) grows smaller, thereby reducing the issuer's expected revenue. When this is the case, facing a more informed liquidity supplier hurts the issuer's ability to raise liquid funds. □

Appendix D: Proof of Theorem 3

Proof. Let $\tilde{F}$ be an arbitrary experiment. Define $\psi_{\tilde{F}}\left[D\right]$ as the issuer's revenue when she proposes a menu of incentive compatible debt contracts characterized by $\left\{D\left(\omega\right)\right\}_{\omega \in \Omega}$. By virtue of lemma 4, this means that

$$\psi_{\tilde{F}}\left[D\right] = \int_\Omega \left(\int_{\mathbb{R}_+} u\left(\min\left\{y, D\left(\omega\right)\right\}, \omega\right) \left(1 - \left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right) \left(\frac{\frac{\partial}{\partial \omega} f\left(y|\omega\right)}{f\left(y|\omega\right)}\right)\right) \mathrm{d}F\left(y|\omega\right)\right) \mathrm{d}\Phi\left(\omega\right).$$

We show that the function $\psi_{\tilde{F}}\left[D\right]$ has the *single crossing differences* property in $\left(D, \tilde{F}\right)$. That is, we show that, for any $D'' > D'$, and any $F'' \succeq_{\text{Lehmann}} F'$,

$$\psi_{F'}\left[D''\right] - \psi_{F'}\left[D'\right] \leq 0 \Rightarrow \psi_{F''}\left[D''\right] - \psi_{F''}\left[D'\right] \leq 0.$$

To see this, first note that

$$\begin{aligned}\frac{\partial}{\partial D(\omega)}\psi_F[D] &= \phi(\omega)(1-F(D(\omega)|\omega)) - (1-\Phi(\omega))\frac{\partial}{\partial\omega}(1-F(D(\omega)|\omega)) \\ &= -\frac{\partial}{\partial\omega}((1-\Phi(\omega))(1-F(D(\omega)|\omega))). \end{aligned} \tag{27}$$

Next, let $\boldsymbol{y}''$ (resp. $\boldsymbol{y}'$) be the cashflows obtained from drawing $\boldsymbol{\omega}$ from Φ and then applying the experiment F'' (resp. F'). Claim (1) in proposition 2 implies that the induced marginal distributions of $\boldsymbol{y}''$ and $\boldsymbol{y}'$ coincide and equals Ψ_Φ. The fact that $F'' \succeq_{\text{Lehmann}} F'$ implies that, for all $z \in [0,1]$, [29]

$$\frac{\frac{\partial}{\partial\omega}f''(\omega|\Psi_\Phi(\boldsymbol{y}'') \geq z)}{f''(\omega|\Psi_\Phi(\boldsymbol{y}'') \geq z)} \geq \frac{\frac{\partial}{\partial\omega}f'(\omega|\Psi_\Phi(\boldsymbol{y}') \geq z)}{f'(\omega|\Psi_\Phi(\boldsymbol{y}') \geq z)},$$

or, equivalently,

$$\frac{\frac{\partial}{\partial\omega}\{\Pr\{\Psi_\Phi(\boldsymbol{y}'') \geq z|\boldsymbol{\omega}=\omega\}\phi(\omega)\}}{\Pr\{\Psi_\Phi(\boldsymbol{y}'') \geq z|\boldsymbol{\omega}=\omega\}\phi(\omega)} \geq \frac{\frac{\partial}{\partial\omega}\{\Pr\{\Psi_\Phi(\boldsymbol{y}') \geq z|\boldsymbol{\omega}=\omega\}\phi(\omega)\}}{\Pr\{\Psi_\Phi(\boldsymbol{y}') \geq z|\boldsymbol{\omega}=\omega\}\phi(\omega)}.$$

Next, note that

$$\frac{\frac{\partial}{\partial\omega}\{\Pr\{\Psi_\Phi(\boldsymbol{y}'') \geq z|\boldsymbol{\omega}=\omega\}\phi(\omega)\}}{\Pr\{\Psi_\Phi(\boldsymbol{y}'') \geq z|\boldsymbol{\omega}=\omega\}\phi(\omega)} = \frac{\frac{\partial}{\partial\omega}\left(1-F''\left(\Psi_\Phi^{-1}(z)|\omega\right)\right)}{1-F''\left(\Psi_\Phi^{-1}(z)|\omega\right)} + \frac{\frac{\mathrm{d}}{\mathrm{d}\omega}\phi(\omega)}{\phi(\omega)},$$

and similarly,

$$\frac{\frac{\partial}{\partial\omega}\{\Pr\{\Psi_\Phi(\boldsymbol{y}') \geq z|\boldsymbol{\omega}=\omega\}\phi(\omega)\}}{\Pr\{\Psi_\Phi(\boldsymbol{y}') \geq z|\boldsymbol{\omega}=\omega\}\phi(\omega)} = \frac{\frac{\partial}{\partial\omega}\left(1-F'\left(\Psi_\Phi^{-1}(z)|\omega\right)\right)}{1-F'\left(\Psi_\Phi^{-1}(z)|\omega\right)} + \frac{\frac{\mathrm{d}}{\mathrm{d}\omega}\phi(\omega)}{\phi(\omega)},$$

Thus, we must have that, for all $z \in [0,1]$,

$$\frac{\frac{\partial}{\partial\omega}\left(1-F''\left(\Psi_\Phi^{-1}(z)|\omega\right)\right)}{1-F''\left(\Psi_\Phi^{-1}(z)|\omega\right)} \geq \frac{\frac{\partial}{\partial\omega}\left(1-F'\left(\Psi_\Phi^{-1}(z)|\omega\right)\right)}{1-F'\left(\Psi_\Phi^{-1}(z)|\omega\right)}. \tag{28}$$

Finally, suppose that for some mechanism characterized by $D(\cdot)$, $\frac{\partial}{\partial D(\omega)}\psi_{F'}[D] \leq 0$. From (27), this is equivalent to having

$$\frac{\phi(\omega)}{1-\Phi(\omega)} \leq \frac{\frac{\partial}{\partial\omega}(1-F'(D(\omega)|\omega))}{1-F'(D(\omega)|\omega)}.$$

[29] See corollary 1 in Athey and Levin [2018].

Inequality (28) then implies that necessarily $\frac{\partial}{\partial D(\omega)}\psi_{F''}[D] \leq 0$. Further, note that, under assumptions 2 and 3, for any experiment $\tilde{F}$, the optimal mechanism $D_*\left(\cdot;\tilde{F}\right)$ is determined by pointwise maximization. The result then follows from Milgrom and Shannon [1994]. □

References

Susan Athey and Jonathan Levin. The value of information in monotone decision problems. *Research in Economics*, 72(1):101–116, 2018.

Ulf Axelson. Security design with investor private information. *The journal of finance*, 62(6): 2587–2632, 2007.

Laurie Simon Bagwell. Shareholder heterogeneity: Evidence and implications. *The American Economic Review*, 81(2):218–221, 1991.

Laurie Simon Bagwell. Dutch auction repurchases: An analysis of shareholder heterogeneity. *The Journal of Finance*, 47(1):71–105, 1992.

Antonio E Bernardo and Bradford Cornell. The valuation of complex derivatives by major investment firms: Empirical evidence. *The Journal of Finance*, 52(2):785–798, 1997.

Bruno Biais and Thomas Mariotti. Strategic liquidity supply and security design. *The Review of Economic Studies*, 72(3):615–649, 2005.

Michael Bradley, Anand Desai, and E Han Kim. Synergistic gains from corporate acquisitions and their division between the stockholders of target and acquiring firms. *Journal of financial Economics*, 21(1):3–40, 1988.

Mike Burkart and Samuel Lee. Smart buyers. *The Review of Corporate Finance Studies*, 5 (2):239–270, 2016.

Jacques Crémer and Richard P McLean. Full extraction of the surplus in bayesian and dominant strategy auctions. *Econometrica: Journal of the Econometric Society*, pages 1247–1257, 1988.

Jacques Crémer et al. Auctions with contingent payments: Comment. *American Economic Review*, 77(4):746–746, 1987.

Brendan Daley, Brett Green, and Victoria Vanasco. Designing securities for scrutiny. *Working paper, John Hopkins University*, 2022.

Peter DeMarzo and Darrell Duffie. A liquidity-based model of security design. *Econometrica*, 67(1):65–99, 1999.

Peter M DeMarzo and Michael J Fishman. Optimal long-term financial contracting. *The Review of Financial Studies*, 20(6):2079–2128, 2007.

Peter M DeMarzo, Ilan Kremer, and Andrzej Skrzypacz. Bidding with securities: Auctions and security design. *American economic review*, 95(4):936–959, 2005.

Mathias Dewatripont, Ian Jewitt, and Jean Tirole. The economics of career concerns, part i: Comparing information structures. *The Review of Economic Studies*, 66(1):183–198, 1999.

Michael Ewens and Matthew Rhodes-Kropf. Is a vc partnership greater than the sum of its partners? *The Journal of Finance*, 70(3):1081–1113, 2015.

Michael Ewens, Ramana Nanda, and Matthew Rhodes-Kropf. Cost of experimentation and the evolution of venture capital. *Journal of Financial Economics*, 128(3):422–442, 2018.

Drew Fudenberg and Jean Tirole. Game theory mit press. *Cambridge, MA*, 86, 1991.

Juan-José Ganuza and Jose S Penalva. Signal orderings based on dispersion and the supply of private information in auctions. *Econometrica*, 78(3):1007–1030, 2010.

Gary Gorton and George Pennacchi. Financial intermediaries and liquidity creation. *The Journal of Finance*, 45(1):49–71, 1990.

Thomas Hellmann and Manju Puri. Venture capital and the professionalization of start-up firms: Empirical evidence. *The journal of finance*, 57(1):169–197, 2002.

Yael V Hochberg, Alexander Ljungqvist, and Yang Lu. Whom you know matters: Venture capital networks and investment performance. *The Journal of Finance*, 62(1):251–301, 2007.

Nicolas Inostroza and Anton Tsoy. Optimal information and security design. *WP, University of Toronto*, 2022.

Ian Jewitt. Information order in decision and agency problems. *mimeo. Oxford University*, 2007.

Steven N Kaplan and Antoinette Schoar. Private equity performance: Returns, persistence, and capital flows. *The journal of finance*, 60(4):1791–1823, 2005.

Son Ku Kim. Efficiency of an information system in an agency model. *Econometrica: Journal of the Econometric Society*, pages 89–102, 1995.

Yonggyun Kim. Comparing information in general monotone decision problems. *Working Paper, Duke*, 2022.

Seokwoo Lee and Uday Rajan. Robust security design. *Ross School of Business, University of Michigan*, (1338), 2018.

EL Lehmann. Comparing location experiments. *Annals of statistics*, 16(2):521–533, 1988.

Hayne E Leland and David H Pyle. Informational asymmetries, financial structure, and financial intermediation. *The journal of Finance*, 32(2):371–387, 1977.

Josh Lerner and Ramana Nanda. Venture capital's role in financing innovation: What we know and how much we still need to learn. *Journal of Economic Perspectives*, 34(3):237–61, 2020.

Tingjun Liu. Optimal equity auctions with heterogeneous bidders. *Journal of Economic Theory*, 166:94–123, 2016.

Tingjun Liu and Dan Bernhardt. Rent extraction with securities plus cash. *The Journal of Finance*, 76(4):1869–1912, 2021.

Andrey Malenko and Anton Tsoy. Asymmetric information and security design under knightian uncertainty. *Mimeo. Ross School of Business, University of Michigan*, 2020.

Teddy Mekonnen and René Leal Vizcaíno. Bayesian comparative statics. *Theoretical Economics*, 17(1):219–251, 2022.

Margaret Meyer and Bruno Strulovici. Increasing interdependence of multivariate distributions. *Journal of Economic Theory*, 147(4):1460–1489, 2012.

Paul Milgrom and Ilya Segal. Envelope theorems for arbitrary choice sets. *Econometrica*, 70 (2):583–601, 2002.

Paul Milgrom and Chris Shannon. Monotone comparative statics. *Econometrica: Journal of the Econometric Society*, pages 157–180, 1994.

Paul R Milgrom and Robert J Weber. A theory of auctions and competitive bidding. *Econometrica: Journal of the Econometric Society*, pages 1089–1122, 1982.

Alfred Müller and Dietrich Stoyan. *Comparison methods for stochastic models and risks*, volume 389. Wiley, 2002.

Michael Mussa and Sherwin Rosen. Monopoly and product quality. *Journal of Economic theory*, 18(2):301–317, 1978.

Stewart C Myers and Nicholas S Majluf. Corporate financing and investment decisions when firms have information that investors do not have. *Journal of Financial Economics*, 13(2): 187–221, 1984.

Roger B Myerson. Optimal auction design. *Mathematics of operations research*, 6(1):58–73, 1981.

David C Nachman and Thomas H Noe. Optimal design of securities under asymmetric information. *The Review of Financial Studies*, 7(1):1–44, 1994.

Marco Ottaviani and Andrea Prat. The value of public information in monopoly. *Econometrica*, 69(6):1673–1683, 2001.

Nicola Persico. Information acquisition in auctions. *Econometrica*, 68(1):135–148, 2000.

John K-H Quah and Bruno Strulovici. Comparative statics, informativeness, and the interval dominance order. *Econometrica*, 77(6):1949–1992, 2009.

Stephen A Ross. The determination of financial structure: the incentive-signalling approach. *The bell journal of economics*, pages 23–40, 1977.

Moshe Shaked and J George Shanthikumar. *Stochastic orders*. Springer, 2007.

Morten Sorensen. How smart is smart money. a twosided matching model of venture capital. *The Journal of Finance*, 62(6):2725–2762, 2007.

Martin Szydlowski. Optimal financing and disclosure. *Management Science*, 67(1):436–454, 2021.

André H Tchen. Inequalities for distributions with given marginals. *The Annals of Probability*, pages 814–827, 1980.

Tereza Tykvová. When and why do venture-capital-backed companies obtain venture lending? *Journal of Financial and Quantitative Analysis*, 52(3):1049–1080, 2017.

Victoria Vanasco. The downside of asset screening for market liquidity. *The Journal of Finance*, 72(5):1937–1982, 2017.

Ming Yang. Optimality of debt under flexible information acquisition. *The Review of Economic Studies*, 87(1):487–536, 2020.

Yue Yuan. Competing with security design. Technical report, Working paper, London School of Economics, 2020.